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HARMONIC FREQUENCY RELATIONS IN THE INFRARED ABSORPTION SPECTRA OF LIQUIDS AND SOLIDS

By Joseph W. Ellis

A self-recording photographic mechanism for securing absorption spectra in the near infrared region has been described by the writer.[1] Data and curves for the absorption spectra of some thirty organic liquids were given to 2.5μ. It was pointed out in this article that absorption bands at approximately 1.7μ, 1.38μ, 1.17μ, 1.02μ and 0.9μ were found in all carbon-hydrogen compounds examined; it was further mentioned that these appeared to be respectively the third to the seventh harmonics of a fundamental wave-length appearing near 6.9μ in Coblentz'[2] absorption spectra of organic compounds. The thickness of the cells was approximately 11 mm.

The investigation has been continued using both very thin films of several of the liquids previously studied and of some new substances; and also cells of about 10 cm thickness. Using the thin films of absorbing material, a band near 2.5μ appeared, previously obscured by the opacity of the thicker cells. This band is assumed to be the second harmonic in the series mentioned above. Using 10 cm cells, two additional bands appeared consistently near 0.83μ and 0.76μ. These are assumed to be respectively the eighth and ninth harmonics in the series.

The relationship throughout this series is not truly harmonic; rather do the frequencies of these absorption maxima seem to be determined by the formula

$$\nu_n = n\nu_0(1 - nx) = n\nu_0 - n^2\nu_0 x \qquad (1)$$

[1] Ellis, Physical Review (In press).
[2] Coblentz, Pub. of the Carnegie Institution, No. 35; 1905.

where n has the values 1, 2, 3, 4 . . . for the fundamental frequency, the first, second, third, etc., harmonics. ν_0 is the constant fundamental frequency and x is a small correction factor. This formula accounts for the progressive deviation toward a longer wave length in the higher frequencies of the series. Kratzer[3] suggested a formula identical with (1) as being applicable to the approximately harmonic relationships in the absorption spectra of the diatomic gases, HCl, HBr and HF. Schaefer and Thomas[4] have extended the series of absorption bands in CO gas. The frequencies of bands observed by them at 4.67μ, 2.35μ and 1.573μ, they point out, satisfy the ratio

$$\nu_0(1-x):2\nu_0(1-2x):3\nu_0(1-3x)$$

rather than the simple ratio 1:2:3.

Curve B of Fig. 1 represents the observed frequencies for a particular substance, cotton-seed oil, plotted against their respective harmonic numbers. The plotted points on Curve A represent values of $n\nu_0$ where the values of ν_0 were calculated separately from each of the eight equations of the form (1). To do this the values of $\nu_0 x$ were calculated from every possible pair of the above equations with the observed frequencies substituted for ν_n. The average value of $\nu_0 x$ was then taken. Curve A is a straight line. Similar curves could have been plotted for the other organic substances examined.

Table 1 gives the data for seven organic substances studied: cotton-seed oil, lard oil, medicinal oil (a pure petroleum distillate), kerosene, mesitylene, pentane and heptane. These data include the observed frequencies, calculated values of ν_0, and the average values of $\nu_0 x$.

The values for the fundamental wave-length obtained by the extrapolation of the above data lie near 6.7μ for all the substances mentioned. Although there are no data in this region of the spectrum for these particular substances, the values for this fundamental wave-length in other organic compounds as given in Coblentz' data are slightly greater than 6.7μ. This would suggest a possible systematic error in this region.

Coblentz[5] gives the following values for absorption maxima in lard oil: 0.7μ, 0.9μ, 1.18μ, 1.41μ and 1.75μ. For cotton-seed oil he gives: 1.18μ, 1.39μ and 1.75μ. Gibson[6] has made a study of the infra-red absorption of certain oils to 1.36μ, using a glass spectroscope of

[3] Kratzer, ZS. f. Physik, *3*, p. 189; 1920.
[4] Schaefer and Thomas, ZS. f. Physik, *12*, p. 330; 1923.
[5] Coblentz, Scientific Papers of the Bur. Stand., No. 418; 1921.
[6] Gibson, Cotton Oil Press, Washington, D. C., Sept. 1920.

fairly high dispersion. He gives the following values for absorption bands in cotton-seed oil: 0.76μ, 0.83μ, 0.93μ, 1.04μ and 1.21μ. These values of Coblentz and Gibson agree well with those of the present investigation. This research has shown, however, that these bands are not characteristic of oils alone, but are due to characteristic oscillations of the *C-H* bond.

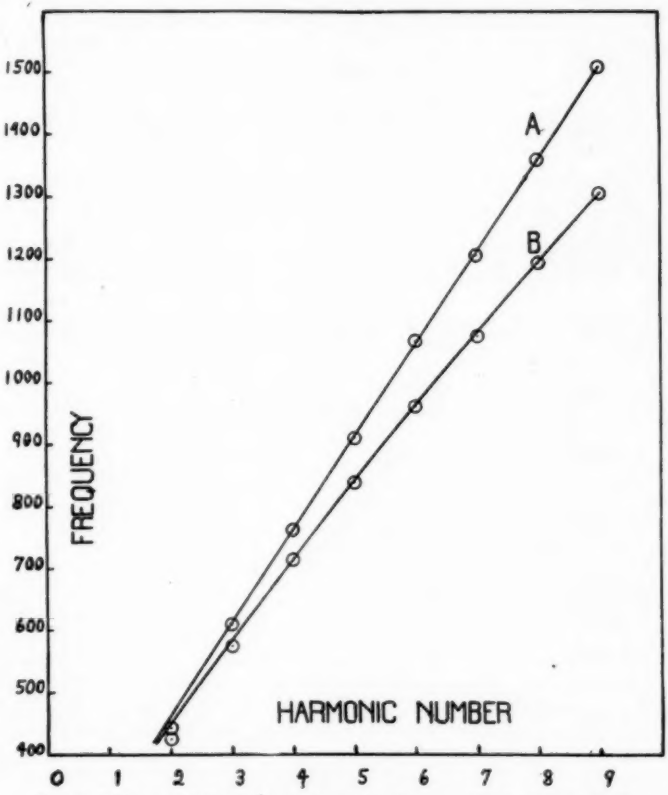

FIG. 1. *Characteristic infrared absorption frequencies of cotton-seed oil*

Water is known to have absorption bands near 6μ, 3μ, 2μ, 1.5μ, 1.2μ and 1μ. The first two have also been observed in many of the alcohols. Coblentz[7] found them in crystals containing water of crystallization and water of constitution. Using 10 cm cells of water the writer

[7] Coblentz, Bulletin, Bureau of Standards, 7; 1911.

has succeeded in clearly locating two additional bands at 0.845μ and 0.745μ. *Water vapor* is known to have a corresponding band for each band given above, including the two new ones, shifted, however, in each case a few per cent toward a shorter wave-length. Thus, the appearance of the bands at 0.845μ and 0.745μ is not surprising, even though the latter lies within the visible red.

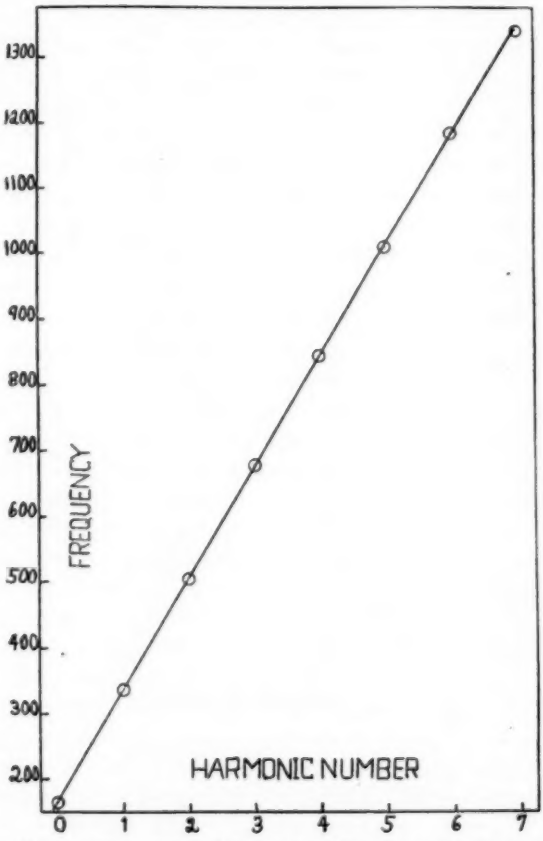

FIG. 2. *Characteristic infrared absorption frequencies of water*

Using the values of the frequencies obtained in the present investigation for the four highest frequencies, and the average of all the values obtainable for the other four frequencies, it is observed that the

seven highest frequencies are harmonics of the frequency whose wavelength is at 6μ. When the harmonic numbers are plotted against observed frequencies, an almost exact linear relationship is obtained. Fig. 2 shows this relationship. The writer's values for the bands,

TABLE 1.—*Harmonic frequencies in the infrared*

Cotton-seed oil					Lard oil				
Harmonic	λ	ν	ν_0	$\nu_0\Sigma$	Harmonic	λ	ν	ν_0	$\nu_0\Sigma$
2	2.35μ	425	148.0		2	2.345	427	148.7	
3	1.74	575	152.0		3	1.73	578	153.0	
4	1.40	715	152.8		4	1.39	719	154.6	
5	1.195	837	151.8		5	1.19	840	153.4	
6	1.04	962	152.3		6	1.035	967	153.0	
7	0.93	1075	150.8		7	0.93	1075	151.6	
8	0.835	1195	151.1		8	0.835	1195	152.1	
9	0.765	1305	151.1		9	0.765	1305	151.5	
				2.05					2.15

Medicinal oil					Kerosene				
Harmonic	λ	ν	ν_0	$\nu_0\Sigma$	Harmonic	λ	ν	ν_0	$\nu_0\Sigma$
2	2.33	429	150.0		2	2.33	429	150.0	
3	1.715	584	155.2		3	1.715	584	155.2	
4	1.37	729	157.4		4	1.37	729	157.4	
5	1.19	840	153.8		5	1.185	845	154.8	
6	1.035	966	154.1		6	1.035	966	154.5	
7	0.93	1075	152.8		7	0.93	1075	153.0	
8	0.835	1195	153.4		8	0.835	1195	153.8	
9	0.76	1315	154.5		9	0.76	1315	154.8	
				2.30					2.30

Mesitylene					Pentane				
Harmonic	λ	ν	ν_0	$\nu_0\Sigma$	Harmonic	λ	ν	ν_0	$\nu_0\Sigma$
2	2.325	430	148.7		2	2.325	430	149.0	
3	1.705	587	154.0		3	1.715	583	153.2	
4	1.37	730	155.0		4	1.375	727	155.0	
5	1.18	848	152.3		5	1.185	844	152.0	
6	1.015	985	153.4		6	1.03	971	152.0	
7	0.915	1092	151.0		7	0.93	1075	149.9	
8	0.825	1211	150.9		8	0.835	1195	149.9	
9	0.750	1333	151.5		9	0.755	1323	151.3	
				1.82					1.90

TABLE 1. *Harmonic frequencies in the infrared* (continued)

Heptane

Harmonic	λ	ν	ν_0	$\nu_0 x$
2	2.35	425	148.0	
3	1.72	582	154.1	
4	1.38	725	155.6	
5	1.19	840	152.7	
6	1.035	967	152.8	
7	0.93	1075	151.1	
8	0.835	1195	151.7	
9	0.765	1305	151.5	
				2.10

1.44μ and 1.96μ, are thought to be too small. It is probable that their positions in the spectrum have been shifted due to a blending of the absorption bands with those due to water vapor in the atmosphere, which always appear in an ordinary intensity distribution curve. No precaution had been taken against this. In this same way we might account for the widely varying values given by different investigators for these bands. The apparent harmonic relationship among the 1.5μ, 3μ and 6μ bands has been commented upon by Kemble.[8]

Table 2 gives the data from which the curve of Fig. 2 was plotted. The values 6.08μ, 2.97μ, 1.98μ and 1.473μ are the average of all the values tabulated in Table 3.

TABLE 2. *Infrared absorption bands of water.*

Harmonic number	λ	ν	ν_0
0	6.08μ	164.5	164.5
1	2.97	337.	168.5
2	1.98	505	168.3
3	1.473	679	169.7
4	1.185	845	169.0
5	0.990	1010	168.3
6	0.845	1184	169.1
7	0.745	1342	168.9

Coblentz[7] also found a band at 4.7μ, which, because of its intensity and position in the spectrum, has no place in the series mentioned above. Coblentz' curve also shows the fundamental and the first harmonic bands to be of practically equal intensities. It also shows the

[8] Kemble, Phys. Rev. *2*, p. 8; 1916.

3μ band displaced too far toward the longer wave-lengths. (His value, 2.95μ, in the preceding table was given by him as determined with the use of his large spectrograph.) The writer has observed that the 4.7μ band has the same position in the spectrum that one of the two characteristic bands of oxygen has. The presence of an O-O bond in water would thus satisfactorily explain the presence of the 4.7μ band and also the shifting and deepening of the 3μ band, since the other band found in oxygen is at 3.2μ. Mr. E. A. Hagman of the College of Chemis-

TABLE 3

Investigator	Year	λ			
Paschen	1894	6.06μ	2.97μ	2.05μ	1.51μ
Aschkinass	1895		3.03	1.96	1.50
Donath	1896			1.96	1.45
Coblentz	1910	6.10	2.95	1.95	1.48
Grantham	1921			1.98	1.48
Collins	1921			2.00	1.44
Ellis	1923			1.96	1.44
Average		6.08μ	2.97μ	1.98μ	1.473μ

try has suggested that this O-O bond might be expected because of the polymerization of water, the formula for water being in general $H_{2n}O_n$. A certain percentage of the water molecules are then represented

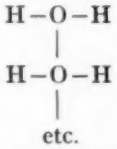

Fig. 3 shows the reproduction of some of the spectral curves. Curves *A*, *B* and *C* respectively show the absorption in a thin film, a 1 mm cell and a 10 cm cell of water. Curves *D* and *E* represent the absorption in 7 cm and 11 mm samples of the medicinal oil. The absorption in a 10 cm cell of heptane is given by Curve *F*.

Coblentz[7] examined the absorption of a number of sulfates and found bands near 9μ and 4.5μ to be characteristic of the sulfate radical. Recently Schaefer and Thomas[4] have extended this investigation. In the case of $SrSO_4$ they found bands at 9.05μ, 4.5μ, 3.02μ and 2.25μ which form an approximate harmonic series. In the case of celestite they found bands at 4.44μ and 2.975μ. These appear to be the first and second harmonics of a band (unobtained) near 8.9μ.

In the present research the absorption of a 2 mm sample of gypsum, $CaSO_4.2H_2O$, has been recorded. Distinct bands at 1.96μ, 1.44μ and

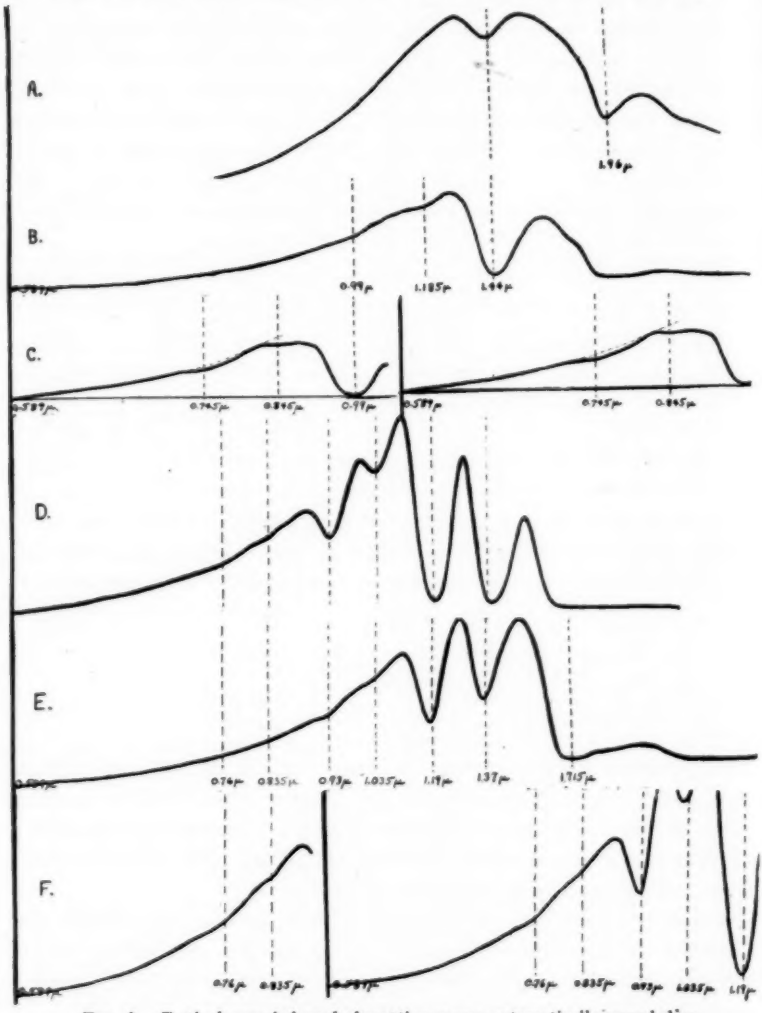

FIG. 3. *Typical near infrared absorption curves automatically recorded on photographic paper.*

1.185μ are doubtless due to the vibrations in the O-H radical of the water molecules, inasmuch as these values check with those of water

in the present study. Two additional bands at 1.77μ and 1.48μ must be attributed to the sulfate radical; these are probably the fourth and fifth harmonics of a fundamental band to be expected near 8.9μ. These bands are probably due to characteristic S-O oscillations. It is of interest to note in the case of the Coblentz curve that SO_2 has a deep band at 8.7μ.

Thus, one is led to conclude that a pair of atoms held in chemical union, such as carbon and hydrogen, oxygen and hydrogen, etc., even though forming a part of a more complex molecule, has not only a fundamental period of vibration to which it responds when an electromagnetic wave of that frequency strikes it; but that it will also respond to whole multiples, or approximately whole multiples, of that frequency under the proper stimulus.

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Emission of Radiation by a Gas, due to Absorption of Radiation by another Gas Mixed with it.—Mercury vapor is mixed with cadmium vapor, and illuminated by wave length 2536.7, absorbed by the mercury but not by the cadmium atoms. Many lines of the cadmium spectrum are emitted; it is supposed that mercury atoms, excited by absorbing quanta of wave length 2536.7, retain the quanta and remain in the state of excitation until they collide with Cd atoms, to which they deliver over the energy of the quantum. Some of the observed Cd lines require, however, a higher excitation-energy than the quantum possesses; one triplet, for example, requires 6.3 equivalent volts, 1.4 more than the quantum has; the excess is supposed to come from translational kinetic energy of the atoms involved in the collision. The same thing occurs with mixtures of mercury vapor and thallium vapor. Certain thallium lines having a much lower excitation-energy than the quantum possesses, are unexpectedly bright in the emission-spectrum; for example the line 3776, requiring only 3.6 equivalent volts, 1.3 less than the quantum has. It is suggested that the 1.3-volt excess goes into translational kinetic energy of the Tl atoms, resulting in a Doppler effect so great that the emitted light is not subject to absorption by nearby unexcited Tl atoms. There is also evidence for a similar effect in the excitation of the sodium D lines by wave length 3303. In this process the electron is lifted from the ($1s$) orbit to the ($3p$) orbit, from which it must go to the ($2p$) orbit before the D-line can be emitted; if the energy liberated in this transition goes into kinetic energy of the atom, the subsequently emitted D line will be displaced by Doppler effect and will not be absorbed to the usual degree by sodium vapor; this is observed. [G. Cario and J. Franck, (Göttingen) ZS. f. Physik, *17*, pp. 202-212; 1923.] K. K. DARROW

The Absorption of Light by Sodium Vapor.—Light emitted by an arc-discharge in the vapor above sodium-potassium alloy (in the spectrum of such a discharge, "the sodium lines always predominate unless the partial pressure of the potassium is many times greater than that of the sodium") was sent through a quartz tube containing sodium and heated by an electric heater. When the sodium vapor in the second tube was not subjected to any electrical stimulation, it absorbed the *D*-lines but not the lines of the subordinate series; this indicated that the sodium atoms were practically all in the normal state, with the valence electron in the $1s$ orbit. When an arc was struck in the second tube, the light passing through the faintly-luminous vapor away from the core of the arc showed absorption not only of the *D*-lines but of the subordinate series lines as well, indicating that a perceptible portion of the sodium atoms are at any moment in an abnormal state, with the valence electron in the $2p$ orbit. Inserting sodium-potassium alloy in the second tube and striking an arc discharge in the vapor, it is found that the subordinate series lines of both metals are absorbed in the excited vapor. [F. H. Newman (Exeter) Phil. Mag., *46*, pp. 420-425; 1923.] KARL K. DARROW

THE PRESENT STATUS OF THE CONSTANTS AND VERIFICATION OF THE LAWS OF THERMAL RADIATION OF A UNIFORMLY HEATED ENCLOSURE*

By W. W. Coblentz

I. Introduction

A preceding article on this subject[1] gave a critical discussion o experimental work completed prior to 1920.

The object of the present note is to summarize recent determinations of the constants of radiation and discuss their bearing upon the laws of radiation.

As indicated in the preceding paper, one of the difficulties in determining the constants of radiation has been the lack of knowledge of the numerical values of the auxiliary constants, such as for example (1) the refractive indices of quartz, fluorite, etc., (2) the temperature scale, and (3) atmospheric absorption. Those laboratories which were not equipped to determine the refractive indices or the temperature scale were obliged to depend upon other laboratories for these data. Moreover, those engaged in establishing the temperature scale depended, in turn, upon the radiation measurements for assistance in verifying their measurements, at high temperatures.

Hence, the present day investigator, looking over the work of the past 20 years, sees not only the progress made in increasing the accuracy of the radiation constants but also of the auxiliary constants.

II. The Constant of Total Radiation

In the preceding report it was shown that the various determinations of the coefficient or so-called Stefan Boltzmann constant, σ, of total radiation, obtained by widely different methods, after applying corrections (principally for atmospheric absorption) give a value of $\sigma = 5.72$ to $5.73 \cdot 10^{-5}$ erg cm^{-2} deg^{-4} sec^{-1}.

In view of the fact that some of the subsequent writers have raised the question of the blackness (completeness of the absorption) of the radiometer receiver, it is relevant to recall that at least one investigator

* Written at the request of the Committee of Spectroradiometry, O. S. A. W. W. Coblentz, Chairman.

[1] This Journal, *5*, p. 131; March, 1921.

Coblentz,[2] determined experimentally the loss of radiation by diffuse reflection from some of the receivers actually used in his experiments. His value of $\sigma = 5.722$ is one of the few which has not undergone any correction. If he had made a correction for atmospheric absorption, of CO_2, amounting to about 0.1, per cent, the highest value at present assignable is $\sigma = 5.73$ which is the average of all the available data.

Recently Wachsmuth and Vierheller[3] have made a determination of the Stefan-Boltzmann constant of radiation. A hollow sphere, 2 cm in diameter, is suspended at the center of metal enclosure 8 cm in diameter, which is evacuated. Both spheres are blackened. The outer one is kept at a constant temperature and the inner one is heated electrically so as to maintain it at a constant temperature. The temperature of the inner sphere ranged from 210°C to 300°C. Various corrections were applied, amounting to 20 to 25 per cent of the total energy supplied. The value obtained is $\sigma = 5.729$ to $5.730.10^{-5}$ erg cm^{-2} deg^{-4}.

An important new determination of the constant of total radiation was recently made by Hoffman[4] using the method of Westphal.[5]

The experimental procedure consists in comparing the emissivity of a cylindrical block of copper, when it is highly polished and when it is blackened, with the emissivity of a black body at the same temperature. As used by Westphal the novelty in the method consisted in having the black body contained within the cylinder. This was probably a defect in operation, owing to a temperature gradient between the black body and the outer surface of the copper cylinders. Westphal obtained a value of $\sigma = 5.57$.

Hoffman introduced modifications. The emissivity of the bright cylinder was compared with that of a black body at a known temperature, using a thermopile. For a black paint he used a mixture of lamp black and waterglass, which he found had a higher emissivity than lamp black for long wave lengths at 184°C.

He obtained 24 sets of observations at temperatures ranging from 90°C to 218°C, the values of the coefficient of radiation ranging from $\sigma = 5.72$ to 5.80. His average value is $\sigma = 5.764 \pm 0.52.10^{-12}$ watt cm^{-2} deg^{-4}, thus showing that this method gives the same value as that of other methods; e.g., of Gerlach and of Coblentz. Hoffmann expresses the opinion that Westphal's original value, $\sigma = 5.57$, is probably in

[2] B. S. Bulletin *12*, p. 553; 1916.
[3] Wachsmuth and Vierheller, Verh. d. Deutsch Phys. Gesell., *2*, p. 36; 1921.
[4] Hoffman. ZS. für Physik, *14*, p. 301; 1923.
[5] Westphal. Verh. d. Deutsch. Phys. Gesell., *14*, p. 987; 1914; *15*, p. 897; 1913.

error owing to an improper determination of the emissivity, caused by temperature differences between the surface layer of the cylinder and the black body.

These data are of special interest in showing how slight variations in procedure may affect the results to be obtained by what appears to be one of the most trustworthy methods for determining this constant.

The new data seem to substantiate the previous conclusions that the value of the coefficient of total radiation is of the order of $\sigma = 5.73. 10^{-12}$ erg cm^{-2} deg^{-4}. Furthermore these new data strengthen the opinion expressed in the previous report that there is little likelihood that the numerical value of the constants of radiation will be swayed by a single and perhaps novel experiment.

III. THE CONSTANT OF SPECTRAL RADIATION

The determination of the coefficient of total radiation by Hoffman was conducted under the direction of the late Prof. Rubens. It seems fitting that his career should have terminated in this problem as well as in a further verification of Planck's spectral radiation formula.

The most recent contribution to the proof of Planck's radiation formula was made by Rubens and Michel.[6] They used four radiators, at temperatures ranging from 16°C to 1558°C. For this temperature range they obtained the isochromatic radiation intensities at wave lengths, 4, 5, 7, 9, 12, 16, 22 and 52μ. For this purpose the various wave lengths were isolated by means of fluorite, rock salt and sylvite prisms, also by means of residual rays reflected from fluorite and from rock salt. In their calculations they adopted the value of the spectral radiation constant $C = 14300$. They found that within the errors of observation (viz. 1 per cent) Planck's formula represents their energy measurements. On the other hand the Nernst-Wulf[7] correction to Planck's formula introduced a systematic deviation which attains a maximum value of 6 per cent.

These results represent something more than the by-product of a student's "Arbeit." They are strengthened by some 30 years experience in radiation work. In a previous similar investigation, Rubens and Kurlbaum,[8] using the residual rays of fluorite and of rock salt, at 22μ

[6] Rubens and Michel, Verh. Preuss. Akad. Wiss; *38*, p. 590; 1921.
[7] Nernst and Wulf, Verh. d. Deutsch. Phys. Gesell., *21*, p. 294; 1919.
[8] Rubens and Kurlbaum, Ann. der Phys. (4), *4*, p. 649; 1901.

and 52μ respectively, found their data in agreement (within 1 per cent) with those calculated by Plank's equation.

As shown in the previous report, the radiation intensity measurements, made by the writer,[9] in the spectral region extending from 1μ to 7μ, are in agreement, within the errors of observation (0.5 to 1 per cent), with the values calculated by means of Planck's formula.

The recent work substantiates the previous conclusion that, within the experimental errors of observation, Planck's equation represents the energy distribution of a black body in the spectrum extending from 0.5μ to 50μ.

As for the questions raised by Nernst and Wulf, concerning the numerical value of the constant, C, obtained from the isothermal spectral energy measurements, it is gratifying to point out that the fault is not found primarily in the radiometric measurements, but in the exactness of the auxiliary constants, especially in the exactness of the refractive indices of the prism material used. For example, by using the most recent and probably the most accurate determination of the refractive index of fluorite for the yellow helium line (used in his zero setting of his spectrobolometer) which is smaller by 3 units in the 5th decimal place than the value previously used, the writer's[10] old value of $C=14369$ is reduced by 0.3 to 0.4 per cent, giving a value of $C=14{,}311$ to $C=14{,}326$.

Similarly owing to very small uncertainties in the refractive indices of the quartz prism used, the values of the spectral radiation constant, obtained by Warburg and his collaborators lies between $C=14{,}300$ and $C=14{,}400$. As mentioned in the previous report, Warburg[11] adopted the round number, $C=14{,}300$, as best representing all his investigations, which were made with quartz and with fluorite prisms.

Recently Michel[12] obtained further isochromatics, using the above mentioned apparatus of Rubens and Michel. No high accuracy is claimed for the exact determination of C, the aim being to prove Planck's formula for a determined magnitude of λT. His value is $C=14{,}270\pm 0.18\%$; though the errors may amount to 0.5 per cent.

The average of the data summarized in the previous paper gave a value of $C=14{,}320$ micron degrees, which is practically the theoretical value. The recent determination does not necessitate changing

[9] Coblentz, B. S. Bul., *16*, 459; 1916.
[10] Coblentz, B. S. Bul., *17*, p. 42; 1920.
[11] Warburg, Verh. d. Deutsch Phys. Gesell., *18*, p. 1; June, 1916.
[12] Michel, ZS. für. Phys., *9*, p. 285; 1922.

the previous estimate of the probable value of this constant. Present day experimental errors preclude determining whether the value is $C = 14{,}300$ or $C = 14{,}320$.

In conclusion, its seems gratifying to leave the constants of radiation practically as previously given; viz.

$$C = 14{,}320 \text{ micron degrees}$$

$$\sigma = 5.72 \cdot 10^{-5} \text{ erg cm}^{-2} \text{ sec}^{-1} \text{ deg}^{-4}$$

BUREAU OF STANDARDS,
WASHINGTON, D. C.

A New Method for Studying Ionizing Potentials.—This is essentially a combination of the familiar Franck-Hertz method of determining ionizing potentials with Thomson's method of positive ray analysis. Electrons are accelerated from a hot filament to a grid G, by a potential-rise V_1, and driven back between G_1 and a second grid G_2 by a potential-fall $-V_1$. Midway between G_1 and G_2 a beam of mercury atoms, coming from a heated reservoir and made as nearly as possible a parallel beam by filtering it through two diaphragms in a long tube with cooled walls, is shot transversely across the path of the electrons; such atoms as are ionized by electrons are drawn through G_2 and then by a second accelerating potential-drop V_3 into a slit, beyond which they are deflected by a magnetic field after the classical method of positive ray analysis. In many measurements V_3 was from ten to thirty times as great as V_1, minimizing the relative variations in the true speeds of the ions in the positive ray chamber; in others, not much greater than V_1. The method is not yet a very delicate one; however, the peaks corresponding to the ions Hg^+ and Hg^{++} are very prominent in all curves taken with $V_1 = 30$ or more; also peaks at $m/e = 15$ and 30 (on the scale of $m/e = 1$ for hydrogen) due probably to air, and with very high values of V, a peak at $m/e = 65$ perhaps due to Hg^{+++}. To determine at just what voltage Hg^{++} first appeared, Smyth determined the threshold-voltage for Hg^+; found it somewhat greater than 10.4, the known value, and assumed the difference to be a constant correction-term (due mainly to the stream of mercury atoms being some distance beyond G_1 where the potential was lower) and subtracted it from the observed threshold values for Hg^{++}. From a large number of curves taken at voltages V_1 between 15 and 30 he concludes that Hg^{++} appears at 19 ± 2 volts. The series limit of the enhanced spectrum for Hg is not known, but judging from the spectra of Cd and Zn must be about 20 volts. [H. D. Smyth (Cavendish) Proc. Roy. Soc. *102A*, pp. 283-293; 1922-1923.]

KARL K. DARROW

INNER QUANTUM NUMBERS FOR THE NEUTRAL HELIUM ATOM*

By Arthur E. Ruark, Paul D. Foote, and F. L. Mohler

The narrow doublets of helium owe their origin to a two-fold p term. Contrary to the state of affairs in the alkali doublet spectra, the short-λ components of the sharp series are stronger than their companions. For this reason the $2p$ level with the larger wave number is called $2p_1$ whereas in the alkalies $2p_2$ has the greater wave number.

Similar relations are found in the diffuse series of the doublet system. The first pair in this series consists of a strong line D_3, λ5875.618, and its companion, frequently called a satellite, at λ5875.960. In analogy with the alkalies one should expect to find each composite doublet of the diffuse series of helium to consist of three members. Nutting[1] found, with an echelon of high resolving power, that D_3 showed traces of resolution. Lohmann,[2] using a 32 step echelon with one centimeter plates, concluded that there is a second satellite of slightly greater wave length than D_3, as shown in Fig. 1*A*. The energy diagram and allocation of inner quantum numbers to which this discovery leads are given in Fig. 1*B*.

Sommerfeld's intensity rule and the selection principle for inner quantum numbers are obeyed in all detail by the observed pattern for $2p-3d$, as well as by all other lines of the doublet spectra. The composite doublet $2p-3d$ consists of the lines $2p_1-3d_1$, $2p_2-3d_2$, $2p_1-3d_2$, listed in the order of decreasing intensity. The first and third lines, on account of the extremely small separation of the d terms, differ so little in wave number that they are usually observed as a single line. The fourth member, $2p_2-3d_1$, is ruled out by the selection principle for the inner quantum number j. The inversion of p levels is not known in other doublet spectra, although it is commonly observed in multiplet spectra. Inversion of the $3d$ levels has been noted by Fowler in Mg^+, and Runge[3] and Paschen[4] have interpreted certain data on potassium as pointing to the existence of inverse

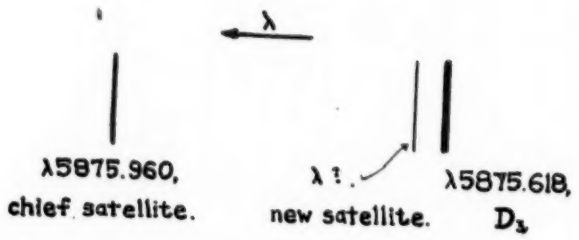
* Published by permission of the Director of the Bureau of Standards of the U. S. Department of Commerce.

[1] Astrophysical Journal, *23*, p. 64; 1906.

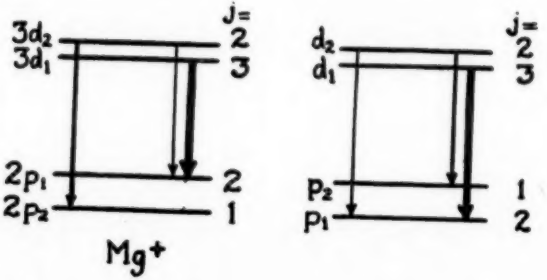
[2] ZS. für wiss. Photographie, *6*, pp. 1 and 41; 1906

[3] Naturwissenschaften, *11*, p. 433; 1923.

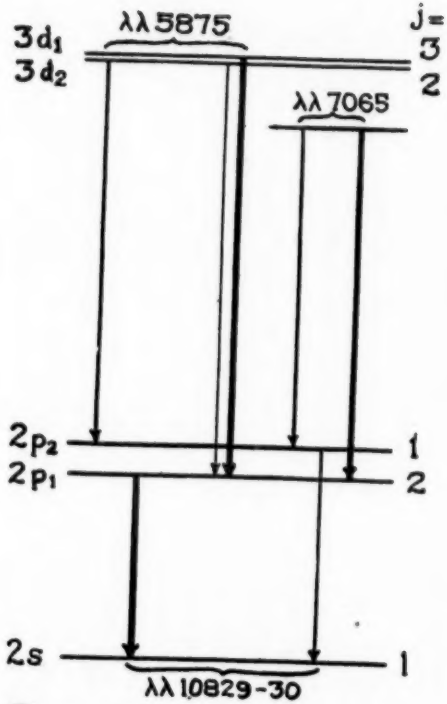
[4] Naturwisserschaften, *11*, p. 434; 1923.



A. The D_3 doublet.



C. Inverse d-levels, D. p and d both inverse.



B. Doublet Levels of Helium.

FIG. 1

$3d$ levels in that element. The energy diagram appropriate to inverse $3d$ levels is shown in Fig. 1*C* while the case, not as yet observed, where both $2p$ and $3d$ levels are inverse is illustrated by Fig. 1*D*.

In assigning inner quantum numbers to the doublet helium lines we commence with the level $2s$. Since the atom has only two electrons and both are on orbits of azimuthal quantum number not greater than 1 in the s states, the largest possible inner number for these states is 2. We exclude the value $j=0$ for the ms states because the mp_1 and mp_2 orbits would then have the same inner number, 1. We cannot decide between the values $j=1$ and $j=2$ for the s states by means of the data at hand.

Sommerfeld[5] has applied the name "uneigentliche" to the helium doublets because they do not show the type of Paschen-Back effect characteristic of the alkali doublets. He states, on the basis of some unpublished work of Back, that certain doublet lines maintain their separations in very strong magnetic fields. However, Paschen and Back[6] have shown that the Zeeman patterns of $2p-3d$ and $2p-4d$ change in character as the magnetic field is increased. So far as can be seen from their data the final result for both doublets would be a normal triplet with its middle component in the position of the line $2p_1-md_1$. In view of these facts there seems to be no experimental reason why the doublet levels of helium should not be assigned inner quantum numbers. In spite of the agreement with selection principles and intensity rules attained in this way, we cannot be certain that Lohmann's observation was not due to echelon ghosts, and it is very desirable that the work should be repeated with crossed echelons.

BUREAU OF STANDARDS.

Low Velocity Secondary Electrons Excited by X-Rays.—The electrons investigated are those emitted from the emergence side of a sheet of gold leaf, irradiated by a beam of X-rays from a Coolidge tube operated at 44000 volts peak voltage, incident on a small area near the center of the sheet. Opposite and parallel to the sheet at a distance of 1 mm was a plane of graphite (spread over a sheet of filter paper); it was charged to various electron-retarding potentials, and the leak of electrons from the gold leaf was measured with an electroscope. Eighty-five per cent of the electrons emitted have initial energies less than 2 equivalent volts; the distribution in energy over the range from 0.3 to 1.4 equivalent volts conforms with the

[5] Atombau, 3rd edition, p. 468.

[6] Annalen d. Phys., *39*, 897; 1912.

Maxwell distribution, like that of thermionic electrons, corresponding however to a very high temperature (about 11000°); for lower energies there are fewer electrons, for higher energies also fewer electrons, than the Maxwell distribution would allow. These slow electrons are probably excited by the fast electrons which are immediately produced by the X-rays. The device for keeping the potential of the gold leaf constant during the efflux of electrons consists of a metal sleeve, surrounding part of the wire leading from the gold leaf to the electroscope, and connected to the rider of a slide wire potentiometer, which is so displaced during the leak as to keep the deflection of the electroscope at zero; after a one minute run the sleeve is earthed, and the immediately occurring deflection of the electroscope gives the total leak during the period. [L. Simons, Birkbeck College (London) Phil. Mag., *46*, pp. 473-480; 1923.]

KARL K. DARROW

The Yvon Spectrophotometer.—While designated by the name spectrophotometer, this apparatus, recently exhibited by A. Jobin, France, constitutes a veritable laboratory for photometry in general. Fourteen different principal combinations of the apparatus are possible as follows:

1. Ordinary homochromatic photometry.
2. Homochromatic absorption photometry.
3. Ordinary microphotometry of absorption.
4. Ordinary absorption and diffusion photometry.
5. Polarimetry.
6. Constant deviation wave length spectometer.
7. Homochromatic spectrophotometer.
8. Absorption spectrophotometer.
9. Absorption microspectrophotometer.
10. Absorption and diffusion spectrophotometer.
11. Spectropolarimeter.
12. Heterochromatic photometer.
13. Heterochromatic absorption photometer.
14. Heterochromatic spectrophotometer.

The parts of the apparatus—some fixed, some movable or pivoted—are arranged on a base plate 58×50 cm in an original and ingenious manner. [M. Yvon, Revue d'Optique, *1*, pp. 499-507; 1922.]

G. W. MOFFITT

OPTICAL SOCIETY OF AMERICA

PROCEEDINGS OF EIGHTH ANNUAL MEETING

CLEVELAND, OHIO

October 26-27, 1923.

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MINUTES OF THE SOCIETY

October 25, 26, 27, 1923

The Eighth Annual Meeting of the Optical Society of America, Dr. L. T. Troland, President, was held at Cleveland, Ohio, Thursday, Friday and Saturday, October 25-27, 1923. Hotel headquarters were at the Hotel Cleveland. All sessions for the reading of papers were held in the Physics Building, Case School of Applied Science.

The meeting was held under the auspices of the following Local Committee in Cleveland:

Representing the National Lamp Works:
Dr. W. E. Forsythe, *Chairman*
Mr. L. C. Kent
Mr. C. D. Spencer
Mr. M. Luckiesh
Mr. A. H. Taylor
Dr. A. G. Worthing
Representing Case School of Applied Science:
Prof. D. C. Miller
Representing Western Reserve University:
Prof. H. W. Mountcastle
Representing Warner and Swasey:
Mr. Warner Seely

In concluding its sessions the Society tendered a most hearty vote of thanks to this committee as well as to the National Lamp Works, Case School of Applied Science, Western Reserve University, Warner and Swasey and the Cleveland Museum of Art for their efforts which resulted in a meeting generally admitted to be the most notable and successful in the history of the Society.

About 50 persons attending the convention registered and obtained rooms at the Hotel Cleveland. The *registered* attendance at Case School was 78, of which 57 were from outside of Cleveland. The *actual* attendance was undoubtedly much greater than this. The number present at the sessions varied from about 50 to over 250.

SPECIAL FEATURES OF THE MEETING:

The address of the retiring President, Dr. L. T. Troland, October 26 was on "The Optics of the Nervous System."

Other notable features of the meeting deserve special mention.

(1) *Prof. A. A. Michelson's paper on "The Limit of Accuracy in Optical Measurement" contributed by invitation on October 26.* In introducing Prof. Michelson, Prof. D. C. Miller of Case School recalled in a very happy manner Prof. Michelson's early connection with the Department of Physics at Case, mentioning his work on the velocity of light, the interferometer, and the renowned experiment on "ether drift." He also exhibited as mementos of this early work parts of Prof. Michelson's original apparatus. Before proceeding with his paper, Prof. Michelson also recounted a number of interesting reminiscences of his first measurements of the velocity of light and the development of the interferometer. Over 250 persons heard Prof. Michelson speak.

(2) *Papers contributed by invitation by Prof. E. L. Nichols as follows on October 26:*

"The Spectral Structure of the Kathodoluminescence of Metals in Solid Solution" by T. Tanaka.

"On the Spectra of Incandescent Oxides" by E. L. Nichols and L. J. Boardman.

(3) *Visits to the Cleveland Museum of Art:*

The Cleveland Museum of Art is located in Wade Park only a short distance from Case School. On October 25, the Director of the Museum, Mr. Frederic Allen Whiting addressed

the meeting by invitation, and explained the work of the museum in a most interesting manner, dwelling particularly on "The Optical Problems of an Art Museum." He extended to all members and guests of the Society a most cordial invitation to visit the Museum. Many availed themselves of this opportunity to visit a museum which is notable and exceptional in many respects and these visits contributed greatly to the pleasure and profit of attendance at the meeting.

(4) *Visit to Nela Park:*

On the afternoon and evening of October 25th, members of the Society were guests of the National Lamp Works at Nela Park. Parties were conducted through the Research Laboratories, the Laboratory of Applied Science, and lamp factories and were given exceptional opportunities to observe the actual manufacture of lamp bulbs and lamps. In the evening a complimentary dinner given to the Society by the National Lamp Works was followed by a Sumposium on Light and Lighting by Prof. E. F. Nichols, Mr. Ward Harrison and Mr. M. Luckiesh and a beautiful experimental demonstration of *The Projection of Mobile Color Patterns* by Messrs. M. Luckiesh and A. H. Taylor of the Nela Laboratory of Applied Science.

(5) *Visit to Warner and Swasey:*

On October 27, the Society visited the plant of Warner and Swasey which is renowned for the construction of the largest astronomical telescope mountings in the world. Members were personally greeted by Mr. Swasey who showed many objects of interest in his office. The mounting for the giant reflector which is just being completed for the Ohio Wesleyan University was on exhibit on the floor of the shop and attracted great interest. The circular dividing engine and various pieces of optical interest made in the shop were also exhibited.

(6) *Inspection of the Laboratories of Physics at Case School of Applied Science.*

On Thursday morning, Prof. Miller welcomed the Society and mentioned the principal apparatus of especial interest in the Case Physical Laboratories. During recesses of the meeting, many members visited the laboratories and inspected the instruments and apparatus.

(7) *Society Dinner:*

The annual dinner, held in the Rose Room of the Hotel Cleveland, on the evening of October 26th was attended by seventy-six persons and was a most enjoyable occasion. President Troland was toastmaster and the following speakers responded: Mr. Charles Brush, Prof. A. A. Michelson, Prof. D. C. Miller, Prof. H. W. Mountcastle, Prof. A. D. Cole, Prof. Frank Allen, Prof. C. A. Skinner, Dr. Hermann Kellner, Dr. Herbert E. Ives.

BUSINESS

The business meeting was held on October 25th.

The results of the election of officers for 1924-5 were declared by the President as follows:

President—Herbert E. Ives

Vice-President—W. E. Forsythe

Members of the Executive Council—K. T. Compton, Theodore Lyman, P. G. Nutting, Fred E. Wright. (Prof. Lyman later declined to serve; and Carl W. Keuffel, being fifth choice on the preferential ballot, was declared elected.)

Informal reports of the Secretary, the Treasurer, the Assistant Editor and Business Manager, and the Committee on Preparing and Publishing an English Translation of Helmholtz' Physiologic Optics were accepted, it being understood that formal reports would be published later at suitable dates.

Brief oral reports of the following Progress Committees were presented and accepted:

Committee	*Chairman*
Colorimetry	E. A. Weaver (Report presented by Troland)
Pyrometry	C. O. Fairchild
Refractometry	I. C. Gardner
Spectrophotometry	K. S. Gibson
Visual Sensitometry	H. M. Johnson

ABSTRACTS OF PAPERS COMMUNICATED TO THE MEETING

Sessions for the reading of papers were held on October 26 and 27. Abstracts of papers as submitted by the authors are given below:

1. G. W. Moffitt and Paul B. Taylor — **Optical Laboratory, Frankford Arsenal**

"THE MEASUREMENT OF TRANSMISSION IN OPTICAL INSTRUMENTS."

The methods described in a number of recent papers are briefly reviewed and discussed. In all these cases results are obtained by comparing the brightness of entrance and exit pupils when one of these is illuminated by a diffusely radiating source.

From theoretical considerations one would expect such a method to give transmission values too high, especially in instruments having considerable flare and scatter. In order to obtain results free from this objection, and to compare methods, apparatus was arranged to measure the brightness of a distant small object on a dark field as seen through the instrument under test and also without it. The two methods give results that are appreciably different in instruments of low transmission.

Transmission in different parts of the field may also be measured by the new method.

(Received Sept. 7, 1923)

2. Wilmer Souder — **Bureau of Standards**

"CONTINUOUS MOTION TO THE DIVIDING ENGINE CARRIAGE."

The usual procedure, when ruling surfaces with a dividing engine, is to move the carriage a certain distance, stop the carriage and cut the line.

There are two elements of error entering to a greater or lesser degree in such procedure:

(a) If the spacing is regulated by the number of teeth or clicks on a ratchet wheel, there is always a factor of wear on the teeth used most. Any change to other intervals may include these worn teeth and give periodic errors.

(b) The tendency for the carriage to creep beyond the position desired (the position at which the driving mechanism is stopped) and the tendency for the carriage to delay starting after having stopped appear to be quite irregular. This variation has been studied by using an interferometer having one plate on the carriage and one on the bed of the engine.

By arranging for continuous motion of the carriage and setting the ruling stroke at the proper angle of lead it is possible to rule lines at right angles to the carriage motion and, apparently, of better regularity of spacing. The lantern slide will show the apparatus more clearly.

(Received Aug. 21, 1923)

(Paper No. 2 was presented by Dr. F. M. Walters, Jr. in the absence of the author)

3. Loyd A. Jones — **Eastman Research Laboratory**

"THE 'CONTRAST' OF DEVELOPING-OUT PAPERS."

From the standpoint of the use of photographic papers, that characteristic commonly referred to as "contrast" is of utmost importance. At the present time, contrast is specified qualitatively by such words as soft, hard, special, vigorous, etc. Obviously, these words can not constitute a precise specification and they become especially confusing when the same words are applied by different manufacturers to papers widely different in quality. A method of numerically specifying this characteristic is desirable.

Complete information relative to the quality of a paper is contained in its characteristic curve, and by proper treatment of the data obtained therefrom it should be possible to evaluate numerically the contrast. This depends upon several factors such as the maximum black, the slope of the straight line portion, total scale, latitude, etc. By

means of a statistical method which is entirely independent of sensitometric measurements, the contrast values of thirty-seven selected printing papers are obtained. By correlation of these values with the sensitometric constants, a formula is derived expressing contrast directly in terms of values determined sensitometrically.

(Received Sept. 8, 1923)

4. T. Townsend Smith — **University of Nebraska**

"COLOR CORRECTION IN IMAGE FORMATION."

Hartmann's interpolation formula is assumed to represent satisfactorily the variation of index of refraction with wave-length within the visible spectrum for a pair of typical telescope glasses. The secondary spectra resulting when different pairs of wave-lengths are achromatized are determined. An expression for the brightness of image as a function of the residual chromatic aberration is developed. It is shown that it is slightly more desirable to achromatize a telescope for the *F* and *B* rather than for the *F* and *C* spectrum lines. The brightest image of an axial object will not be at the focal point for the wave-length of minimum focal length but at a point on the axis slightly more distant from the lens.

Bibliography:
Wilsing: Zeits. Instrumentenk. *26*, p. 41; 1906.
Strehl; Zeits. Instrumentenk. *17*, p. 50; 1897,

(Received Sept. 10, 1923)

(Paper No. 4 was presented by Dr. I. C. Gardner in the absence of the author)

5. L. B. Tuckerman — **Bureau of Standards**

"THEORY OF THE OPTICAL LEVER AND A NEW OPTICAL LEVER SYSTEM."

The general law of multiple reflections at a system of plane mirrors is stated. A convenient quaternion formula for expressing the relative rotation of object and image space is given. The relative position is not conveniently expressible.

A collimated and especially an autocollimated observing system is much freer from accidental disturbance than an uncollimated system.

In certain arrangements of mirror directions the change of space rotation is a simple function of the mirror rotation. These are convenient for optical lever systems.

Special Systems and their properties

1 Single Mirror
2 Double Mirror
3 Triple Mirror—A New Arrangement
4 Combined Double and Triple Mirror

Description of an Optical Strain Gage using Triple Mirror System.

Bibliography:
Georg Gehlhoff: "Das Autocollimationsablesefernrohr." Zeitschrift f. Tech. Phys. vol. 3, p. 225; 1922.

(Received Sept. 8, 1923)

6. Ludwik Silberstein — **Eastman Research Laboratory**

"ASPHERICAL LENS SYSTEMS."

The properties of the well-known Cartesian surfaces being briefly recalled, a general method is given for constructing an aberrationless system of any number of such refracting surfaces, each of these latter introducing a new free parameter. Next, it is shown how such parameters can be utilized for satisfying, in addition to the freedom from aberration, other requirements. As an example a method is worked out for satisfying, by two Cartesian surfaces, the sine-condition, up to fourth order terms, and thus constructing what is technically called an aplanatic lens. Lastly, the achromatization of

such a lens by the insertion of a third surface is dealt with in general terms and illustrated by a numerical example.

The present paper will appear in full in J. O S. A. and R. S I.

(Received Sept. 8, 1923)

7. Ludwik Silberstein — **Eastman Research Laboratory**

"OPTICAL COLLINEATION, INDEPENDENT OF METRICS."

The purpose of the paper is to show how the *optical collineation* (representation) of Möbius-Abbe can be treated without any reference whatever to metrical concepts such as distance or angle, in fine, by purely projective geometry. A graphical definition of the collineation (or straight-edge construction of image to any given object) being set up, all the fundamental formulas are deduced therefrom by the use of non-metrical or projective vector algebra, implying vector addition only. These formulas are a non-metrical generalization of the familiar ones, including those for the magnification the role of rigid lengths (inches or centimeters) being taken over by "staudtians" or projectively equal steps.

Bibliography:

Silberstein: "Projective Vector Algebra," London, G. Bell, 1919. (The first few sections only of this book are required for following the paper.)

(Received Sept. 8, 1923)

8. Herbert E. Ives — **Western Electric Co., New York**

"THE BRIGHTNESS OF THE BLACK BODY AT THE MELTING POINT OF PLATINUM."

A hollow cylinder of highly polished platinum, provided with a narrow slit for observation of the interior, constitutes, when viewed at certain angles, a very satisfactory black body. Heated electrically, such cylinders may be observed photometrically up to their melting point. Observations of the brightness of a series of platinum cylinder black bodies at their melting points have been carried out by a photoelectric photometric method. An image of the opening in the cylinder is projected on an aperture over a diffusing glass in front of a photoelectric cell, screened to approximate the response characteristics of the eye, and a record of the brightness, up to the instant of melting, made by the use of a string electrometer and moving photographic film. Similar records of standard lamps permit the determination of the quantity in question. The mean value from a large number of melts is, for the brightness of the black body at the melting point of platinum, *55.4 candle power per square centimeter*, which is believed to be accurate to within ¼ per cent.

The definiteness with which this point may be determined, and the relative simplicity of the apparatus required recommend it for consideration as a *primary fixed point or standard in photometry*, as suggested by Waidner and Burgess.

Bibliography:

Waidner and Burgess: "The Primary Standard of Light," Electrical World, Sept. 19, 1908, p. 625.
Ives: "The Black Body at the Melting Point of Platinum as a Fixed Point in Photometry." Physical Rev., Sept. 1916, p. 250.

(Received Sept. 10, 1923)

9. M. S. Hollenberg — **University of Manitoba**

"ON THE VERIFICATION OF THE PRINCIPLE OF REFLEX VISUAL SENSATIONS."

These experiments were undertaken to verify the reflex visual sensations recently discovered by Professor Allen. The same apparatus and methods were used by the writer as were employed by Allen. The reflex and the fatigue curves obtained are of precisely the same character as Allen's. Only two of the equilibrium colors were tested, viz: 572 and 660 millimicrons, and these behaved as Allen had found.

These results are applied to explain the initial "overshooting" of the physiological intensity of colors (Broca and Sulzer), and also anomalous trichromatic vision.

Bibliography:
Allen: "On Reflex Visual Sensation." J. O. S. A. and R. S. I., August, 1923.

The present paper will appear in full in J. O. S. A. and R. S. I.

(Received Sept. 8, 1923)

(Paper No. 9 was presented by Prof. Frank Allen)

10. Frank Allen **University of Manitoba**

"ON REFLEX VISUAL SENSATIONS AND COLOR CONTRAST."

Following the discovery of reflex visual sensations, published in the August number (1923) of the Jour. Op. Soc., experiments were made to determine whether colors, white light, and darkness, acting on one area of a retina affected the adjoining areas in any way.

Spectral colors of known wave-length were allowed to fall on one side of the vertical median line of the right retina, and it was found that reflex enhancement of the three primary sensations, red, green, and violet, was produced on the other side of the retina, the predominant enhancement being in the portions of the spectrum complementary to the active color. The six equilibrium colors, 660, 570, 520, 505, 480, and 425 millimicrons produced no reflex effects.

White light and darkness similarly enhanced the brightness of the whole spectrum. It was found that the black interior of the eye piece of the spectrometer had an important enhancing effect on the colors under observation.

These experiments afford explanations of simultaneous contrast, colored shadows, luminosity contrast, successive contrast.

The explanation of these phenomena is physiological, and illusions and deceptions of judgment need not be invoked.

Certain deductions are made regarding the probable non-contrasting peculiarities of equilibrium colors.

Bibliography:
Allen: "Reflex Visual Sensations." J. O. S. A. and R. S. I., August, 1923.

The present paper will appear in full in J. O. S. A. and R. S. I.

(Received Sept. 8, 1923)

11. William Mayo Venable

"COLOR AND LUMINOSITY."

There are three elementary sensations of brightness, white, yellow and green. Each of these sensations is capable of excitation negatively, and when so excited diminishes the brightness of the total or resultant sensation, at the same time exciting a chroma antagonistic to that of the positively luminous sensation of which it is the negative or opposite. These three sensations opposite to the three brightness sensations may be called sensations of darkness, or negative white, negative yellow and negative green. The brightness of any color, including the so-called "subjective" colors seen in flicker, after images and colors seen by contrast, are of brightness equal to the algebraic sum of the brightnesses of the elementary colors entering into the color combination. Red is yellow luminosity diminished in brightness by that form of darkness that is the opposite to green; purple is white luminosity diminished by the same "negative green." Blue is white diminished in brightness by "negative yellow."

By applying data relating to complementary wave lengths and hue sensibility to spectral hues, the "Average Relative Visibility Curve" of the spectrum is separated into three component visibility curves representing the elementary color pairs as in the normal eye. It is shown that the various departures of visibility curves from the normal are completely explicable as due to different relative sensibilities for each color pair, not only for approximately normal eyes but also for color blind eyes and eyes fatigued

by dazzling by various spectral hues. An explanation is provided for the spectrum appearing brighter to an eye adapted to diffused daylight than to one accustomed to total darkness, and for the shifting of the wave length of maximum visibility with changes in intensity. A flicker method is demonstrated for measuring contrast sensibility for each of the elementary color pairs separately. Some discussion of the bearings of these relations upon the current theories of color vision is presented, and a few speculations indulged about the possible retinal mechanism.

Bibliography:

Coblentz and Emerson on Relative Sensibility, Scientific Papers No. 303, Bureau of Standards. Nutting on the Reactions of the Retina to Light, and Allen on Persistence of Vision and the Primary Color Sensations both in Am. Journ. Phys. Opt., April 1920. Priest: Jo. Op. Soc. Am., *4*, pp. 402-404; 1920.

The present paper will appear in full in the American Journal of Physiological Optics.

(Received Aug. 14, 1923)

12. Irwin G. Priest **Bureau of Standards and Munsell Research Laboratory**

"APPARATUS FOR THE DETERMINATION OF COLOR IN TERMS OF DOMINANT WAVE-LENGTH, PURITY AND BRIGHTNESS."

Description of an assembly of apparatus consisting of a "monochromatic colorimeter" (previously so-called) and complete accessory apparatus for the determination of dominant wave-length, purity and transmission or reflection for white light under definitely specified conditions.

The apparatus is described as it has been actually used at the Bureau of Standards since March 1923.

Salient features of the apparatus and its use are:—

1) The use of a specified artificial white light, field illumination being from incandescent filament lamps.

2) The diffusion sphere for the diffuse illumination of samples to be examined by reflection.

3) The convenient arrangement for handling samples to be examined in either transmitted or reflected light.

4) The convenient arrangement for the simultaneous adjustment of dominant wave-length, purity and brightness in the mixture field to match the juxtaposed sample field.

5) The analysis of the mixture of homogeneous and white light by a substitution flicker method.

Details of design of the apparatus, explicit directions for its use and data on its performance as regards precision and accuracy, including extensive tests of the additivity of homogeneous and white brightnesses, are given in the complete paper.

Bibliography:

Abney: "Colour Measurement and Mixture." (London 1891), Chap. XIII.

Nutting: B. S. Bull, *9*, pp. 1-5; 1913.

Luckiesh: "Color and its Applications." 2nd Edition, 1921, pp. 99-101.

Ives: J. O. S. A. and R. S. I., 7, p. 291; 1923.

The present paper will appear in full in J. O. S. A. and R. S. I.

(Received Aug 15, 1923)

13. Irwin G. Priest, K. S. Gibson, A. E. O. Munsell
Bureau of Standards and Munsell Research Laboratory

"A COMPARISON OF EXPERIMENTAL VALUES OF DOMINANT WAVE-LENGTH AND PURITY WITH THEIR VALUES COMPUTED FROM THE SPECTRAL DISTRIBUTION OF THE STIMULUS."

Excepting the purples, any color may be matched by a suitable mixture of homogeneous light and white light. The DOMINANT WAVE-LENGTH (λ) is the wave-length of the homogeneous light.

$$p \equiv \text{PURITY} = \left\{ \frac{\text{Homogeneous Light}}{\text{Homogeneous Light} + \text{White Light}} \right\}$$

$$i \equiv \text{IMPURITY} = 1 - p$$

It is possible to compute, by way of the theory of three "elementary excitations," the dominant wave length and purity which should be found experimentally for a color evoked by a stimulus of given spectral distribution.

The following table shows a comparison between values of dominant wave-length and purity determined experimentally by the authors and values computed on two different bases, proposed respectively by the Colorimetry Committee, O. S. A., and by H. E. Ives.

DOMINANT WAVE LENGTH						PURITY					
EXPERIMENTAL*				COMPUTED		EXPERIMENTAL*				COMPUTED	
IGP	KSG	AEOM	Mean	O.S.A. Com.**	Ives***	IGP	KSG	AEOM	Mean	O.S.A. Com.**	Ives***
449.0[a]	446.1[a]	447.0[a]				0.846[b]	0.842[b]	0.819[b]			
448.4[a]	446.0[a]	447.7[a]				.845[b]	.824[b]	.894[b]			
447.6[a]	447.1[a]	447.2[a]				.872[b]	.856[b]	.846[b]			
448.3	446.4	447.3	447.3	450	442	.854	.841	.853	0.849	0.80	0.32
535.8[a]	542.5[a]	536.7[a]				0.761[c]	0.884[c]	0.791[c]			
542.3[a]	540.6[a]	540.6[a]				.915[c]	.846[c]	.861[c]			
535.6[a]	540.3[a]	540.8[a]				.774[c]	.844[c]	.887[c]			
537.9	541.1	539.4	539.5	538.4	539.6	.817	.858	.846	0.840	0.885	0.880
573.9[a]	573.9[a]	573.7[a]				0.824[c]	0.836[c]	0.810[c]			
573.7[a]	573.5[a]	573.6[a]				.842[c]	.829[c]	.809[c]			
573.4[a]	573.7[a]	574.4[a]				.812[c]	.819[c]	.805[c]			
573.7	573.7	573.9	573.8	573.3	573.9	.826	.828	.808	0.821	0.820	0.840
639.3[a]	640.2[a]	639.6[a]				0.99[d]	0.99[d]	0.99[d]			
634.5[a]	636.6[a]	642.0[a]				.99[d]	.99[d]	.99[d]			
646.0[a]	639.4[a]	634.2[a]				.99[d]	.99[d]	.99[d]			
643.6	638.7	638.7	640.3	636.3	638.5	.99	.99	.99	0.99	1.000	1.000
477.5[a]	478.2[a]	477.2[a]				0.148[c]	0.118[c]	0.125[c]			
478.4[a]	478.2[a]	477.3[a]				.166[c]	.137[c]	.132[c]			
478.6[a]	479.2[a]	477.8[a]				.144[c]	.127[c]	.124[c]			
478.2	478.9	477.4	478.2	475.8	475.6	.153	.127	.127	0.136	0.160	0.120
540.1[a]	547.8[a]	538.4[a]				0.281[c]	0.328[c]	0.264[c]			
523.5[a]	540.0[a]	546.2[a]				.199[c]	.264[c]	.309[c]			
532.5[a]	539.9[a]	544.6[a]				.234[c]	.264[c]	.293[c]			
532.0	542.6	543.1	539.2	534.0	529.0	.238	.299	.289	0.275	0.310	0.295
572.4[a]	573.6[a]	572.0[a]				0.267[c]	0.289[c]	0.267[c]			
574.1[a]	572.7[a]	572.4[a]				.268[c]	.278[c]	.292[c]			
572.6[a]	572.6[a]	573.1[a]				.285[c]	.266[c]	.294[c]			
573.0	573.0	572.5	572.6	572.2	571.0	.273	.268	.284	0.282	0.255	0.276
581.5[a]	581.6[a]	581.9[a]				0.250[c]	0.278[c]	0.255[c]			
581.2[a]	580.9[a]	581.2[a]				.220[c]	.247[c]	.221[c]			
581.6[a]	582.3[a]	581.0[a]				.263[c]	.235[c]	.274[c]			
581.4	581.6	581.4	581.5	582.0	578.5	.258	.253	.262	0.258	0.229	0.255

Notes on Standards of White Light

*Gas-filled tungsten lamp with Ives-Brady daylight filter, giving color temperature of 5000°K.

**Noon sun, Mean, table 8, "Report of the Colorimetry Committee," J. O. S. A. and R. S. I., 6, p. 363; 1922.

***Planckian radiator ("black-body") at 5000°K.

Notes on Number of Observation

a. Mean of 10 observations of A, each with independent adjustment of purity and brightness.

b. Mean of 10 measurements of p by collimator slit widths. No measurements of i.

c. Mean of 5 to 10 measurements each of p and i by substitution flicker method.

d. Experimentally, the sample was perfectly matched by the spectrum without adding any white light. Therefore, no purity measurements were made, the experimental purity being obviously equal to 1.00. However, there is about one per cent of "stray" heterogeneous light in the spectrum at this wave length. The purity has, therefore, been entered as 0.99. Further test has shown that this one per cent impurity is less than perceptible as a difference in saturation.

Bibliography:

"Report of the Colorimetry Committee," J. O. S. A and R. S. I., August, 1922.

H. E. Ives: "Transformation of Color Mixture Equations—" J. F. I., Jan., 1923.

Priest: "Apparatus for the Determination of Color—" (preceding paper, this program)

The present paper will appear in full in J. O. S. A. and R. S. I.

(Received Sept. 10, 1923)

(Note. Because of lack of time, papers 12 and 13 were presented together and only very briefly in about five minutes.)

14. Irwin G. Priest, H. J. McNicholas, M. Katherine Frehafer
Bureau of Standards and Munsell Research Laboratory

"SOME TESTS OF THE PRECISION AND RELIABILITY OF MEASUREMENTS OF SPECTRAL TRANSMISSION BY THE KOENIG-MARTENS SPECTROPHOTOMETER."

The Koenig-Martens spectrophotometer as installed and used at the Bureau of Standards has been tested by measuring the transmissions of rotating sectored disks of known angular aperture. Measurements of the spectral transmission of blue filters by different observers have also been compared. The photometric probable error at various wave-lengths has been computed from the observations of three different observers.

From the data, which are presented in considerable detail in tables and graphs, the following conclusions are drawn:

(1) For heterogeneous illumination, the probable error of a single observation is about seven-tenths of one percent of the transmission when measurements are made at the field brightness most favorable to accurate matching.

(2) For homogeneous illumination, the probable error of a single observation of transmission 0.37 is nearly constant and equal to about one percent of the transmission in the case of certain wave-lengths of mercury and helium.

(3) For heterogeneous illumination the actual error (Difference between measured value and true value as given by the angular calibration of the sectored disk) of the mean of 10 observations for transmissions even as low as 0.015 and for wave-lengths between 430 and 700 mμ is usually less than one percent of the transmission.

(4) There is no error of measured transmission systematic with respect to wave length.

Bibliography:
F. F. Martins and F. Grünbaum: Ann. der Phys. (4) 12 pp. 984-1003; 1903.
The present paper will appear in full in J. O. S A. and R S. I.

(Received Sept 10, 1923)

(Note. Because of lack of time, the presentation of paper 14 was omitted on motion of the authors.)

15. Enoch Karrer **Wire Div., Nat. Lamp Wks., G. E. Co.**

"A RATIONAL CGS SYSTEM OF PHOTOMETRIC UNITS."

Light is an objective reality: viz photic radiation. Photometry is a branch of radiometry. The watt is the unit of power in radiometry when the whole power content of the radiation is referred to. In other cases in radiometry where the power of the radiation is measured in reference to some important effect depending upon the wave length, some unit of power in terms of the effect is also basic. This weighted power unit may in all cases be related to the watt in a simple way. It is suggested that such a unit of power for photometry be termed phot, being the photometric power of the radiation at 556 mμ whose total power is one watt. Unit point source of light emits one phot. Other units may then be defined:

Quantity	Unit		Ratio
	cgs	engr.	cgs. to engr
power	phot	candle	53.2
flux	ablumen	lumen	"
brightness	ablambert	lambert (others)	"
illumination	abrumford	lux (many others)	
intensity	abcandle	candle	"
energy	phot-hour	lumen-hour, candlehour	$4\pi \times 53.2$, 53.2

If we ignore the hypothetical point source as a point of departure we may arrive at units that are in one sense simpler:

Quantity	Unit	Symbol	Symb. def.	Equivalent in present engr. system
power	phot	ph		667 lumens (1), $667/4\pi$ candles (c)
brightness	ablambert	abla	ph/cm²	667 1/cm², $667/4\pi$ c/cm²; 667 lamberts (la)
illumination	abrumford	abr	ph/cm²;	667 1/cm²; .0667 lux
intensity	abcandle	abc	ph/strad	667 c; 667 1/steradian
energy	phot-hour	ph-hr		
	or lume	lu	ph×hr	667 1-hr; $667/4\pi$ c−hr

Energy may be referred to as lumenized, photicised, photometric, or lighting energy.

This viewpoint makes the photo-mechanical conversion factor the most fundamental photometrical quantity. Next is the relative photicity curve. These are matters for standardization. The need for a primary standard of light (as a candle) vanishes when such a cgs unit of photometric power is adopted just as the need for a standard of mechanical power (the horse) vanished when the cgs unit of power (the watt) was adopted. The problems of the establishment and maintenance of a primary standard of light become problems of radiometry—the absolute measurement of the power of radiation. Secondary or working standards may be required in either case.

Bibliography:

Eisler: Elektrotechnische Zeitschr., *25*, p, 188, 1904; Ill. Engr. Nomenclature and Standards, Tr. I. E. S., *13*. p. 512, 1918 and *17*, p. 1; 1922.

Karrer: J. Fr. Inst., July, p. 79; 1923.

Crittenden: J. Wash. Acad. of Sci., *13*, p 69; 1923.

Ives, Coblentz and Kingsbury: Phys. Rev., (*2*) *5*, p. 269; 1915.

(Received Aug. 29, 1923)

16. F. E. Ross — **Eastman Research Laboratory**

"DISTORTION OF PHOTOGRAPHIC FILM."

The observation of Cheshire and of Curtis, that photographic film is subject to large and irregular contraction on development is confirmed only in the case of N. C. or non-curling film. For motion picture, aerial, portrait, and process films, the contractions and irregularities are found to be considerably less, in the two last mentioned, only one-tenth (about 0.05%) of those found by Cheshire and Curtis. The changes in length and width are different, the length of a film showing increased effect, due probably to strains in the roll. Films of all kinds are subject to parallel changes in length from day to day and from winter to summer, due to humidity conditions. There is no local distortion in portrait film nor in motion picture and aerial. Local distortions of 10μ sometimes appear in N. C. film on development which however gradually disappear in a few days.

The present paper will appear in full in the Astrophysical Journal.

(Received Sept 8, 1923)

(The author did not respond when paper No. 16 was called and this paper was not read.)

17. W. W. Coblentz — **Bureau of Standards**

"SOME THERMOELECTRICAL PROPERTIES OF MOLYBDENITE."

The present data were obtained in connection with an extensive investigation of the actinoelectrical and photoelectrical properties of molybdenite.

The material tested was in the form of cleavage pieces, split perpendicular to the c-axis, some lamina being 6 cm, or more in length. To the ends were attached fine (#36) copper wires by means of Wood's alloy. The thermal emfs were measured with a potentiometer.

Previously published measurements indicated a thermoelectric power of +750 microvolts (μv). The present measurements on samples from various sources gave values of +690 μv down to +100 μv, through zero, to −180 μv and to −1040 μv. In one sample, one end gave +510 μv and the other gave −1040 μv, against copper, at 28°C. The latter is not unlike a compound thermocouple of copper-iron-bismuth-copper.

Evidently one cannot consider molybdenite of sufficient homogeneity for specifying its thermoelectrical properties.

The present paper will appear in full in Bureau of Standards Scientific Papers.

(Received Sept. 7, 1923)

(Paper No. 17 was presented by Dr. K. S. Gibson in the absence of the author.)

18. Arthur E. Ruark, Paul D. Foote, and F. L. Mohler **Bureau of Standards**

"INNER QUANTUM NUMBERS FOR THE NEUTRAL HELIUM ATOM."

The short wave-length components of the sharp and diffuse He doublets are much stronger than their companions; hence the $2p$ level of larger wave number is called $2p_1$. Nutting and also Lohmann found, with powerful echelons, that the doublet $2p-3d$ contains a second satellite of slightly greater wave-length than that of the intense component. Thus the $3d$ level is double and the composite doublet $2p-3d$ consists of 3 lines. Relative values of the inner quanta j may be assigned as in alkalis except that the $2p$ levels are inverse. On this interpretation Sommerfeld's intensity rule and selection principle for inner quanta are obeyed by all orthohelium lines. Absolute values of j are not known. $j=0$ is excluded for s levels since it requires $j=1$ for both p levels, but j is either 1 or 2 for s levels, since the maximum can not exceed the sum of the azimuthal numbers. If inner quanta really possess the physical significance attributed to them, the average orbital inclinations in the p_1, d_1 etc. states must differ markedly from those in the p_2, d_2 etc. states. This is difficult to reconcile with small doublet separations.

Bibliography:
Nutting: Astrophys. J. *23*, p. 64; 1906.
Lohmann: Z. S. Wiss. Phot., *6*, p. 1, 41; 1908.

(Received Sept. 7, 1923)

19. F. M. Walters, Jr. **Bureau of Standards**

"REGULARITIES IN THE ARC SPECTRUM OF IRON."

In complex spectra many important lines can be arranged in groups involving constant wave number differences, which represent combinations of spectral terms. The number of lines in a group depends on the complexity of the spectral terms, which have been found to possess multiplicity varying from one to eight. Such groups ("multiplets") have recently been discovered in the spectra of manganese, chromium, molybdenum, titanium, scandium, vanadium and iron.

Lines belonging to multiplets have similar characteristics as to intensity, temperature classification and pressure shift. In the arc spectrum of iron there have been found 43 multiplets involving about 500 lines. These include 193 out of 251 lines listed as originating in furnace spectra at a temperature of about 1800°C. The majority of the remaining lines are produced only at much higher temperatures and remain to be classified.

Bibliography:
Catalan: Phil. Trans. Roy. Soc., London A223, p, 127; 1922.
Walters: Journ. Wash. Acad. Sci., *13*, p. 243; 1923.

(Received Sept. 10, 1923)

20. C. Davisson and J. R. Weeks **Research Laboratories of the American Telephone & Telegraph Co., and the Western Electric Co.**

"THE RELATION BETWEEN THE TOTAL THERMAL EMISSIVE POWER OF A METAL AND ITS ELECTRICAL RESISTIVITY."

The electromagnetic theory of light leads to an expression for the coefficient of reflection of a metal which depends upon the wave-length, state of polarization and angle of incidence of the radiation and upon the electrical resistivity of the metal. It follows that the theory must also lead to an expression for the total thermal emissive power of a metal which depends only upon its temperature and resistivity. By Kirchhof's law the reflection coefficient may be converted into a coefficient of emissive power for radiation of like characteristics. And this into the corresponding intensity of black body emission at the same temperature gives the intensity of emission of particular wave-length, polarization and angle of emergence from the metal. Integration over all angles and wave-lengths gives the total emission and this divided by the total black body emission gives finally the total emissive power of the metal.

These integrations have been carried out, partly by graphical methods, and are found to lead to

$$E=0.748(\mathrm{Tr})^{1/2}-0.626(\mathrm{Tr})+0.663(\mathrm{Tr})^{3/2}+\ \ldots\ ,$$

r being the resistivity of the metal in ohm-cms. In making this calculation effects arising from dielectric properties of the metal are neglected and it is assumed that the resistivity of a metal is independent of the frequency of an impressed electric field.

Values of E for platinum have been determined experimentally from observations on the dimensions, temperature and dissipation of power in middle sections (included between potential leads) of straight wires electrically heated in vacuo, corrections being made for conduction losses and for absorption of radiation from surroundings.

At 300°K the observed value of E is in sensible agreement with that calculated from the formula. The ratio of the observed to the calculated value decreases with increasing temperature and passes through a minimum (0.90) at 450°K. The value unity is reached at 700°K and at the highest observed temperature (1500°K) the ratio is 1.12.

The minimum at 450°K appears to be associated with an abnormally high reflecting power of platinum observed by Hagen and Rubens at 6μ. The departures at high temperatures are thought to result from an increase in resistivity of platinum with the frequency of the impressed field.

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Aschkinass: Ann. d. Phys., *17*, pp. 960-976; 1905.
Lummer: E. T. Z., *34*, pp. 1428-1429; 1913.
Foote: B. S. Bull. *11*, pp. 607-612; 1915.
Weber: Ann. d. Phys., (4) *54*, pp. 165-181; 1918.

The present paper will appear in full in J. O. S. A. and R. S. I.

(Received Sept. 10, 1923)

21. Edison Pettit **Mount Wilson Observatory**

"THE EFFECTS OF HEAT ON THE FIGURE OF MIRRORS."

Seven pairs of plane mirrors of varying thicknesses and diameters and of various materials were used in the investigation. The mirrors were placed in the coelostats of the tower telescopes and the Snow horizontal telescope and then exposed to the sun. The position of the focal plane of the projecting system was determined by observing the image of the sun. Of the seven systems used the greatest focal changes were observed in crown glass mirrors one inch thick by twelve inches diameter. The least focal change was observed with the speculum metal mirrors. These were closely rivalled by the pyrex mirrors. These latter mirrors change focus 20 inches during the first hour when

the 150-foot focus lens is used to project the image. The focus then remains fixed throughout the day. All the mirrors became concave upon exposure to the sun except the speculum metal mirrors and the crown glass mirrors of the Snow Horizontal Telescope which became convex. This is an anomaly difficult to explain with any simple theory of expansion in a mirror.

The present paper will appear in full in the Astrophysical Journal.

(Received Sept. 6, 1923)

(Paper No. 21 was presented by Dr. I. C. Gardner in the absence of the author.)

22. Roderick B. Jones and Arthur C. Hardy — **Mass. Institute of Technology**

"A THALOFIDE CELL PYROMETER."

A thalofide cell pyrometer was constructed and compared with an optical pyrometer by making simultaneous readings of the temperature inside an electric furnace. With controlled conditions, the error in measuring the furnace temperature with the thalofide pyrometer was found to be even less than with the optical pyrometer. This precision is due to the fact that, over the range of temperatures investigated, the change in resistance of the thalofide cell is proportional to nearly the seventh power of the absolute temperature. Thus, an unsteadiness in the cell resistance amounting to fourteen per cent introduces an error in the temperature measurement of only two per cent.

Perhaps the only advantage which the thalofide pyrometer has over an optical pyrometer is the possibility of recording temperatures automatically. For many purposes it should be more satisfactory than a radiation pyrometer because the high sensitivity of the thalofide cell permits a smaller angular field to be used.

(Received Sept. 10, 1923)

23. L. V. Foster — **Scientific Bureau, Bausch and Lomb**

"AN IMPROVED METALLURGICAL MICROSCOPE."

A brief review of the standard methods of illumination used in the metallurgical microscope and their difficulties of adjustment will be given. The purpose of the paper will be to describe a new system of illumination which eliminates these adjustments and at the same time makes the metallurgical microscope and illuminating system a simple and compact unit. Improved photographic representation of metal structure results from this system.

(Received Sept. 10, 1923)

24. Hermann Kellner — **Scientific Bureau, Bausch and Lomb**

"A NEW COMPARISON PRISM FOR COLORIMETERS OF THE DUBOSCQ TYPE."

The Duboscq Colorimeter and its comparison prism are briefly described. The modification by Pellin which is an improvement because of its finer dividing line suffers from internal reflections which are great enough to influence the readings. They are avoided by suitable choice of angles in the prisms described in this paper. Because of the delicacy with which matches can be judged the prism may be found useful in other instruments.

The present paper will appear in full in J. O. S. A. and R. S. I.

(Received Sept. 10, 1923)

25. Hermann Kellner — **Scientific Bureau, Bausch and Lomb**

"AN APPARATUS FOR TESTING STRAIN IN GLASS SLABS AND FINISHED PRISMS."

The arrangement of the apparatus is such that the strain figures of the lenses etc. which form part of the apparatus are not superimposed upon the strain figures of the

objects under observation, while the field is so large that all parts of the strain figures of objects of 100 mm diameter can be seen at once.

The present paper will appear in full in J. O. S. A. and R. S. I.

(Received Sept. 10, 1923)

26. **Hermann Kellner** — **Scientific Bureau, Bausch and Lomb**

"A MONOCHROMATOR FOR MERCURY LIGHT."

The monochromator here described was designed for use with Mercury arc lamps, particularly with the Lab-arc.

It may also be used with other light sources, as long as their spectrum shows well separated lines.

Entrance and exit openings for the light lie in the same horizontal axis. The dispersing apparatus is placed in the vertical pillar which means compact construction. A simple, one-piece dispersion prism of novel design is used.

The present paper will appear in full in J. O. S. A. and R. S. I.

(Received Sept. 10, 1923)

27. **G. W. Moffitt** — **Optical Laboratory, Frankford Arsenal**

"CAMERA LENSES OF LARGE RELATIVE APERTURE FOR STELLAR SPECTROG RAPHS."

For the spectroscopy of very faint celestial objects the well known radial velocity type of spectrograph necessitates exposures that are often too long for satisfactory results. In order to decrease the exposure it would be a logical step to concentrate the available light into a spectrum of smaller linear dimensions through the use of a spectrograph camera lens of shorter focal length. But since the collimator dimensions are fixed by considerations too well known to need enumeration here the diameter of the camera lens cannot be reduced in proportion to its folca length. That is, it would be desirable to increase the relative aperture of the camera lens to values of F/3 and larger. At the same time the corrections of the lens must be such as to satisfy the critical demands of the astronomer, and, because of the reduced scale of the spectrum, even more critical correction is required than has heretofore been necessary with ordinary relative apertures.

Since no lens of very large relative aperture hitherto constructed has been found to give satisfactory results as the camera lens of a stellar spectrograph, an investigation was undertaken with a view to providing a lens for this use.

The theoretical requirements are discussed and the characteristics of a suitable lens given in detail.

A lens of 10″ focus and 3″ clear aperture constructed for the Dominion Astrophysical Observatory gave such a good account of itself in actual trial that other observatories have had lenses of approximately similar characteristics designed and constructed.

As a result of these investigations not only has the relative aperture of lenses available for use in spectrograph cameras been greatly increased but in addition the whole of the stellar photographic spectrum can now be sharply imaged on one plate, thereby greatly facilitating the work of the spectroscopist in many cases.

(Received Sept. 7, 1923)

28. **G. W. Moffitt** — **Optical Laboratory, Frankford Arsenal**

"A PRISM FOR SMALL BROKEN TELESCOPES."

The roof angled reflecting prism used in small broken, or elbow, telescopes is generally considered to be a difficult and uncertain prism in manufacture. In order to provide an alternative system for instruments of this class that is a straightforward production proposition a new prism has been evolved.

In optical effect it is similar to the Porro system in that it has a slight tendency to flatten the field of the objective—an advantage not possessed to any appreciable degree by the roof prism.

While somewhat more bulky than the roof prism the new system leads to certain desirable features in housing and adjustment. Its chief advantage, however, lies in its easy manufacture as compared with the difficulties often experienced in the making of roof angled prisms.

(Received Sept. 7, 1923)

29. H. M. Johnson **Ohio State University**

"SPEED, CONSTANCY AND ACCURACY OF RESPONSE TO VISUAL STIMULI AS RELATED TO THE DISTRIBUTION OF BRIGHTNESS."

A report of the results of 14,000 measurements of the time required for discriminative reaction to visual stimuli. The mean brightness of the test-field was kept practically constant at 2.6 millilamberts; and the brightness of the surroundings was varied between 0.02 and 9.5 relatively to the brightness of the field. The stimulus to reaction was the removal of one of two circular spots whose diameters subtended 24′ at the eye of the observer and whose brightness constituted additions of 3.5% to that of the test field.

The results yielded three criteria of performance: namely, speed, constancy, and accuracy.

By all the criteria taken singly or in combination performance was impaired slightly by large reductions of the brightness of the surroundings below that of the test-field, but it was greatly impaired by very moderate increases of their brightness above that of the test-field, the impairment increasing rapidly as such changes progressed.

When appropriately expressed the results are in striking agreement with those obtained by Cobb on the dependence of sensitivity to brightness difference on the distribution of brightnesses. The results clearly indicate the desirability of increased use of auxiliary local lighting in certain types of operations.

To be published in full, with bibliography in the Journal of Experimental Psychology.

(Received after advance program was sent to press. The author did not respond when this paper was called, Oct. 27, and it was not read.)

30. Enoch Karrer **Wire Div., Nat. Lamp Wks., G. E. Co.**

"A VARIABLE SECTOR DISK WITHOUT GEARS READABLE DIRECTLY WITHOUT ANY AUXILIARY OPTICAL OR ELECTRICAL DEVICE."

Two disks are mounted on independent axes of two motors that are driven synchronously. By rotating the field of one of the motors the phase of the disks is changed, i.e., a relative displacement of the disks is effected, and therefore, the relative aperture through the disks is varied. The disks need not necessarily be near each other, but may be widely separated in an optical instrument and placed at convenient positions with the restriction that their images must eventually accurately overlap. For example in the spectrophotometer one may be in a collimator arm and the other in the telescope near the observer and easily accessible for adjusting and for reading.

The preliminary experiments were carried out with aluminum disks mounted upon the shafts of two A. C. 60 cycle 110 volt synchronous motors. Any other type of impulse motors may be used and driven by a tuning fork for instance.

The scale which is actually read to indicate the relative disk aperture may be graduated upon a drum or disk attached directly to the field magnets that are movable. Or, if some electrical means is employed to change the phase of the disks, then the scale that is read may be a scale graduated upon a variable condenser for instance. In the latter case, the disk is readable and adjustable from any distance.

(Received after advance program was sent to press.)

MINUTES OF THE EXECUTIVE COUNCIL

AD INTERIM MINUTES BETWEEN THE SEVENTH ANNUAL MEETING AND THE EIGHTH ANNUAL MEETING.

The following resolution was submitted to vote of the Council by mail ballot June 30, 1923:

"THAT the Editors-in-Chief of the Journal of the Optical Society of America and Review of Scientific Instruments be authorized to offer subscriptions to the Journal en bloc to members of the Physical Society at $3.00 each, provided 500 subscriptions can be obtained, and furthermore that the Editors-in-Chief shall constitute a committee with power to determine the manner in which this transaction shall be carried out and to consummate the same."

Ballots were cast as follows:—

In favor of the resolution:—Lomb, Foote, Ives, Forsythe, Ames, Gale, Richtmyer, Troland.

Opposed to the resolution:—None

Not voting:—Merritt, Southall, Priest.

The President declared the resolution adopted on Aug. 15, 1923.

(*Signed*) Irwin G. Priest,
Secretary.

MINUTES OF THE EXECUTIVE COUNCIL CLEVELAND, OHIO, OCT., 24, 25, 1923.

Two meetings of the Executive Council were held in connection with the annual meeting of the Society at Cleveland. The first meeting was held at the Hotel Cleveland, Oct. 24. The second meeting was held at Nela Park, Oct. 25. The following members were present at both meetings: Troland, Ives, Priest, Lomb, Foote, Richtmyer, Ames and Forsythe.

The following is a digest of the actions taken at these meetings:

(1) The resignations of the following members were accepted:

Regular Members:	
Wheeler P. Davey	December 31, 1922.
F. A. Saunders	January 31, 1923
Associate Members:	
Arthur S. Allen	April, 1923
Roger H. Sinden	April 11, 1923
William H. Williams	August 4, 1923.

(2) The following members were transferred from associate to regular membership:

Elmer E. Hotaling	G. M. J. Mackay
Edison Pettit	Arthur G. Stillhamer
H. B. Williams	Frank Allen
Chas. Bittinger	Charles B. Bazzoni
Wm. J. Drisko	Francis E. Cady
Gordon H. Gliddon	Wilmer Souder
Alfred N. Finn	L. Willis Bugbee, Jr.
John C. Kurtz	J. C. McLennan
Walter Porter White	Raymond T. Birge
Thomas B. Brown	Robert C. Heflebower
John C. Wollensak	Walter Guyton Cady

(3) Prof. F. K. Richtmyer, was elected Representative to the Council of the American Association for the Advancement of Science.

(4) A request received through the Librarian of the Bureau of Standards for a free set of the Journal of the Optical Society of America for the Physikalische-Technische Reichsanstalt, Germany, was referred to the Assistant Editor-in-Chief and Business Manager with power to act.

(5) Troland and Priest were appointed as a committee to select the time and place of the next annual meeting and were instructed to arrange for a meeting in Boston (or Cambridge) Mass. if possible, or in Ithaca, New York if it should not be found practicable to meet in Boston.

(6) The secretary was instructed to designate Mr. Adolph Lomb and the Eastman Kodak Company as "patrons" of the Society in the forthcoming directory.

(7) The secretary was authorized to have old records of the Society bound.

(8) Article V of the By-Laws was amended to read:

"*V-Publications*—This society shall publish a journal devoted to the advancement of optics. The editor-in-chief and the assistant editor-in-chief and business manager of the Journal and six or more associate editors shall be elected by the executive council of the Society. The terms of office of the editor-in-chief and the assistant editor-in-chief and business manager shall be six years, and the expiration of their respective terms shall not coincide. The terms of office of the associate editors shall be six years, one third of them to be elected biennially. Every regular honorary and associate member of the society shall be entitled to receive the Journal."

This amendment was adopted with the proviso that the terms of office of the present incumbents would expire as follows:

P. D. Foote, Editor-in-Chief	Dec. 31, 1925
F. K. Richtmyer, Asst. Editor-in-Chief and Bus. Mgr.	Dec. 31, 1927.

(9) The following were elected associate editors of the *Journal*, each for a term of six years beginning January 1, 1924.

R. A. Millikan	L. P. Sieg
H. S. Uhler	J. P. C. Southall
H. E. Ives	E. C. Crittenden

(10) The following committee was appointed with power to make a new contract with the Association of Scientific Apparatus Makers of the United States of America for the publication of the "Journal of the Optical Society of America and Review of Scientific Instruments."

F. K. Richtmyer, Chairman
Adolph Lomb
P. D. Foote.

(11) An expenditure of $176.89 was authorized for the advance publication of the program of the Eighth Annual Meeting.

(*Signed*) Irwin G. Priest, Secretary.

Approved.
(*Signed*) L. T. Troland,
President

REPORT OF THE SECRETARY

(This report covers business from Oct. 25, 1922 to Oct. 27, 1923 inclusive. It follows the report under date of Oct. 24, 1922, published in the J. O. S. A. and R. S. I., 7, pp. 144-146; Jan. 1923.)

The following errata in the preceding report should be corrected:—

(1) page 144, under losses by resignation, for "17" read "20."
(2) page 144, under total losses, for "19," read "22."
(3) page 144, under "Present membership, Regular," for "258," read "256."
(4) page 144, under "Present membership, Associate Corporation," for "7," read "6."

(5) page 145, in list of losses by resignation, for "H. C. Buck," read "H. E. Buck."
(6) page 145, under list of losses by resignation add the following:—

"E. J. Bissell, Dec. 31, 1921
Motion Picture News, Dec. 31, 1921
A. Albert Klein, Aug. 25, 1922."

The chief activities of the Secretary's office have been: (1) the compilation of the revised membership directory; (2) the preparation for the eighth annual meeting; (3) the recording of the minutes of the seventh annual meeting and the minutes of the executive council; the conduct of the election of officers, October, 1923.

The following items deserve special mention.

I MEMBERSHIP.

A synopsis of the membership is given in the following table.

		Changes Oct. 25, 1922 to Oct. 27, 1923 Inclusive									
		GAINS			LOSSES						
	As of record Oct. 24, 1922 (O)	By election	By transfer	TOTAL (G)	By death	By resignation	By transfer	By non-payment of dues	TOTAL (L)	O+G−L	By Count in new directory
HONORARY	3	2		2						5	5
REGULAR	256	12	22	34	4	2	2	3	11	279	279
REGULAR LIFE	4		2	2						6	6
ASSO. INDIV.	39	53		53		3	22	3	28	64	64
ASSO. CORP.	6	1		1						7	7
									TOTAL	361	361

The following is a detailed statement of membership changes.

1. Elections at the Seventh Annual Meeting.

Honorary

No. 331 A. A. Michelson
No. 332 S. W. Stratton

Regular

No. 333 Horace L. Howes
No. 334 Louis Williams McKeehan
No. 335 Harlan True Stetson
No. 336 Louis Bell
No. 337 Janet Howel Clark
No. 338 Bassett Jones
No. 339 E. J. Brady
No. 340 John Fred Mohler

No. 341 Franklin L. Hunt
No. 342 Otto Stuhlman, Jr. (Effective Jan. 1, 1923)
No. 343 George K. Burgess
No. 344 F. C. Brown

Associate

No. 345 Horatio B. Williams
No. 346 Daniel O. Landis
No. 347 Henry D. Hubbard
No. 348 Harold Frederic Stimson
No. 349 Edison Pettit
No. 350 Roger H. Sinden
No. 351 J. M. McCallie
No. 352 Elmer E. Hotaling
No. 353 Emory Hill
No. 354 Howard O. Stearns

2. *Election to Associate Membership between the Seventh and Eighth Meetings.*

(See By-laws Art. I, Sec. 3)

No. 355 Arthur G. Stillhamer
No. 356 John C. Wollensak
No. 357 Howard W. Weinhart
No. 358 Thomas B. Brown
No. 359 Margaret Montague Monroe
No. 360 L. J. Buttolph
No. 361 Howard McKee Elsey
No. 362 Antonius de Graaff
No. 363 T. S. Sligh, Jr.
No. 364 Walter Porter White
No. 365 William R. Amberson
No. 366 Gordon H. Gliddon
No. 367 Wm. F. Roeser
No. 368 Donald Charles Stockbarger
No. 369 Wm. J. Drisko
No. 370 Ewart S. Taylerson
No. 371 Yu Tai Yao
No. 372 Sherman M. Fairchild
No. 373 G. M. J. Mackay
No. 374 Alfred N. Finn
No. 375 C. Matsuda
No. 376 W. G. Brombacher
No. 377 John C. Kurtz
No. 378 Melville Fuller Peters
No. 379 George H. Rhodes
No. 380 Frank Allen
No. 381 Charles J. Ling
No. 382 Michael Samuel Hollenberg
No. 383 H. E. Marsh
No. 384 Onepiece Bifocal Lens Company
No. 385 Andrew Merritt McMahon
No. 386 Miriam E. O Brien
No. 387 Louise L. Sloan

No. 388 Israel Maizlish
No. 389 William Mayo Venable
No. 390 Charles B. Bazzoni
No. 391 Melville Eastham
No. 392 Francis E. Cady (Effective Jan. 1, 1924)
No. 393 Walter Guyton Cady
No. 394 Wilmer Souder (Effective Jan. 1, 1924)
No. 395 L. Willis Bugbee, Jr.
No. 396 John Cunningham McLennan
No. 397 Raymond T. Birge
No. 398 Robert C. Heflebower

3. *Transfers of Associate Members to Regular Membership by Action of the Council at the Eighth Annual Meeting.*

Elmer E. Hotaling
Edison Pettit
H. B. Williams
Chas. Bittinger
Wm. J. Drisko
Gordon H. Gliddon
Alfred N. Finn
John C. Kurtz
Walter Porter White
Thomas B. Brown
John C. Wollensak
G. M. J. Mackay
Arthur G. Stillhamer
Frank Allen
Charles B. Bazzoni
Francis E. Cady
Wilmer Souder
L. Willis Bugbee, Jr.
J. C. McLennan
Raymond T. Birge
Robert C. Heflebower
Walter Guyton Cady

4. *Transfers to Life Membership.*

(See By-laws Art. II, Sec. 1)

Fred E. Wright
Arthur G. Stillhamer

5. *Resignations Accepted at Eighth Annual Meeting.*

Regular Members:

Wheeler P. Davey	Dec. 31, 1922
F. A. Saunders	Jan. 31, 1923

Associate Members:

Arthur S. Allen	April, 1923
Roger H. Sinden	April 11, 1923
William H. Williams	Aug. 4, 1923

6. Memberships Terminated Because of Non-payment of Dues.

(See "Action Relative to Delinquent Members" J. O. S. A. and R. S. I., 7, p. 211; Feb. 1923.)

Regular Members:

William Churchill
Alfred B. Hitchens
Harry I. Shultz

Associate Members;

H. J. Braddy
H. Brainard Brown
Olaf Tenow

7. Deaths Reported.

Regular Members:

C. W. Waggoner	October, 1922
U. M. Smith	April 21, 1923
Louis Derr	May, 1923
Louis Bell	June 14, 1923

II. PATRONS

(See By-laws Art. II, Sec. 1)

The following being duly qualified, have been designated as Patrons of the Society:—

Adolph Lomb
Eastman Kodak Co.

III. OFFICERS FOR 1924-25.

The following are the officers duly elected and qualified for the calendar years 1924-1925.

President:

Herbert E. Ives

Vice President:

W. E. Forsythe

Secretary:

Irwin G. Priest

Treasurer:

Adolph Lomb

Members of the Executive Council

Ex officio Members:

The above Officers
The Past President, Leonard T. Troland
The Editor-in-Chief of the Journal, Paul D. Foote
The Assistant Editor-in-Chief and Business Manager of the Journal, F. K. Richtmyer

Elected Members:

K. T. Compton
Carl W. Keuffel
P. G. Nutting
F. E. Wright

IV. AMENDMENT TO BY-LAWS.

Article V of the By-laws has been amended to read as follows:—

"V-Publications

This society shall publish a journal devoted to the advancement of optics. The editor-in chief and the assistant editor in-chief and business manager of the Journal and

six or more associate editors shall be elected by the executive council of the society. The terms of office of the editor-in-chief and the assistant editor-in-chief and business manager shall be six years, and the expiration of their respective terms shall not coincide. The terms of office of the associate editors shall be six years, one third of them to be elected biennially. Every regular, honorary, and associate member of the society shall be entitled to receive the Journal."

V. REFERENCE LIST OF OFFICIAL NOTICES PUBLISHED BY THE SECRETARY.

(Continuation of List, J. O. S. A. & R. S. I., 7, p. 146; 1923)

Proceedings, Seventh Annual Meeting:
J. O. S. A. & R. S. I., 7, pp. 59-102; Jan. 1923.
Science, pp. 636-640; Dec. 1, 1922.
Report of the Secretary:
J. O. S. A. & R. S. I., 7, pp. 144-146; Jan. 1923.
Minutes of the Executive Council:
J. O. S. A. & R. S. I., 7, pp. 209-211; Feb. 1923.
Notice of Election of Officers:
J. O. S. A. & R. S. I., 7, p. 304; April, 1923.
Preliminary Announcement of Eighth Annual Meeting:
J. O. S. A. & R. S. I., 7, p. 542; July, 1923. Science, p. 10; July 6, 1923.
Organization of Committees:
J. O. S. A. & R. S. I., 7, pp. 581-582; July, 1923.

The Preliminary Announcement of the Eighth Annual Meeting was mailed to all members, June 16, 1923. It was also sent to the Departments of Physics and Psychology in about thirty Universities and to a number of non-members whose work would indicate their probable interest in the meeting.

(*Signed*) Irwin G. Priest, Secretary.

WASHINGTON,
Nov. 22, 1923.

REPORT OF THE TREASURER

Since the Society's fiscal year ends November 30, the Treasurer presents an estimated statement at this meeting, forecasting as nearly as possible a synopsis of receipts and disbursements, subject to corrections, compared with fiscal year ending November 30, 1922, submitted for the information of the members at this meeting.

	Year Ending *November 30, 1922.* (Actual)	Estimate for Year Ending *November 30, 1923.* (Based upon figures to *July 15, 1923*)
RECEIPTS		
Cash on Hand, Beginning of Year	$ 3,730.61	$ 5,961.63
Dues in Arrears	655.00	835.00
Dues for 1922	1,186.10	1,591.80
Life Membership Dues	50.00	50.00
Dues in Advance	30.00	10.00
Journal Subscriptions from Non-Members	194.25	163.50
Sale of Journals—Back Numbers	789.45	130.90
Sale of Reprints	193.13	.....
Accounts Receivable—Sale of Reprints	211.50	.....

	Year Ending *November 30, 1922.* (Actual)	Estimate for Year Ending *November 30, 1923.* (Based upon figures to *July 15, 1923*)
RECEIPTS (Continued)		
Donations	3,000.00	3,000.00
Interest on Bank Balances	165.22	111.16
Helmholtz Translation Fund (Contra)	1,053.00	.....
Advertising—Current	480.00	450.00
Advertising—Arrears	.....	180.00
Incidentals	30.05	.....
	$11,768.31	$12,483.99
DISBURSEMENTS		
Printing and Stationery	$ 583.77	$ 566.64
Postage and Incidentals	85.56	38.23
Clerical Salaries	35.35	.....
Association of Scientific Apparatus Makers of U. S. (contract)	3,839.00	4,918.00
Helmholtz Translation Fund (Contra)	1,053.00	.....
Translation Rights	200.00	.....
Dues Refunded	10.00	.....
Cash on Hand, End of Year	5,961.63	6,961.12
	$11,768.31	$12,483.99

REPORT OF THE TELLERS ON ELECTION OF OFFICERS FOR THE CALENDAR YEARS 1924-1925.

October 20, 1923.

Dr. L. T. Troland, President,
Optical Society of America,
(Thru Irwin G. Priest, Secretary)

Dear Sir:

We, the undersigned regular members of the Optical Society of America, being duly appointed tellers, have counted the ballots in the election of officers concluded October 19, 1923. We have checked our count and hereby certify that the following are the true results in accord with the rules printed on the ballot:

147 ballots were received. 3 were discarded because they were unsigned. 1 arrived after the time limit set for opening the ballots, leaving 143 ballots to be counted.

(1) Ballots were cast for President as follows:

Herbert E. Ives	94
A. H. Pfund	49
Total	143

(2) Ballots were cast for Vice President as follows:

W. E. Forsythe	83
L. A. Jones	59
No preference	1
Total	143

(3) In the election of the Executive Council, 15 ballots were invalid because no first choice was indicated, and 3 were invalid because more than one first choice was indicated, leaving

125 valid ballots. As a result of the preferential ballot the following were chosen members of the executive council:

P. G. Nutting
Theodore Lyman
K. T. Compton
F. E. Wright

The tally sheet of the election of the executive council is enclosed.

Respectfully submitted,

(*Signed*) L. B. Tuckerman
K. S. Gibson
H. J. McNicholas
Tellers

(The above results were announced by the President at the meeting of the Society October 25, 1923 and the officers-elect were notified by the Secretary. All except Prof. Lyman accepted. Prof. Lyman declined the office because of the pressure of other duties. Carl W. Keuffel, being fifth choice by the preferential ballot was later declared elected to fill the vacancy caused by Prof. Lyman's declining to serve.)

SUPPLEMENT TO TELLERS' REPORT

November 13, 1923.

Dr. L. T. Troland, President,
Optical Society of America,
(Thru Irwin G. Priest, Secretary).

Dear Sir:

Supplementing our report of October 20th, we certify that the fifth choice for members of the executive council was

Carl W. Keuffel

who should therefore be declared elected to fill the vacancy caused by Prof. Lyman's declining to serve.

(*Signed*) L. B. Tuckerman
K. S. Gibson
H. J. McNicholas
Tellers

CONSTITUTION, BY-LAWS AND MEMBERSHIP OF THE OPTICAL SOCIETY OF AMERICA

[November 1, 1923]

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GENERAL INFORMATION

The Optical Society of America was organized in 1916. As stated in its constitution, "It is the aim and purpose of this society to increase and diffuse the knowledge of optics, to promote the mutual interests of investigators of optical problems, of designers, manufacturers and users of optical instruments and apparatus of all kinds and to encourage cooperation among them." While the society pays especial attention to "applied" optics and, on this account, covers a field not previously covered, it is not to be regarded as a technological society in contradistinction to a society devoted to "pure" science. The aim of the Society is to cover the field of optics, including "pure" optics as well as "optical engineering." It solicits the support and membership of all persons "interested in optics" whatever their particular interest may be. The present membership (Nov. 1, 1923) is 361.

The Society has held Annual Meetings as follows:

Date	*Place*	*President*
Dec. 28, 1916	New York, N. Y.	P. G. Nutting.
1917 (Meeting to have been in Pittsburgh cancelled on account of war)		
Dec. 27, 1918	Baltimore, Md.	F. E. Wright.
Feb. 26-27, 1920	*New York, N. Y.	F. E. Wright.
Dec. 27-29, 1920	Chicago, Ill.	F. K. Richtmyer.
Oct. 24-26, 1921	Rochester, N. Y.	J. P. C. Southall.
Oct. 25-28, 1922	Washington	L. T. Troland.
Oct. 25-27, 1923	Cleveland, Ohio	L. T. Troland.

The first number of the Journal of the Optical Society was issued under date of January 1917. The publication was designated as "bi-monthly," but during the war the dates of issue were necessarily irregular and the publication discontinuous. Librarians and others will be interested in the following statement of issues. During the calendar years 1917-19 inclusive there were six separate issues designated as follows:

Vol. I., No. 1, January, 1917.
Vol. I., Nos. 2-3, March-May, 1917.
Vol. I., No. 4, July, 1917.
Vol. I., Nos. 5-6, September-November, 1917.
Vols. II-III., Nos. 1-2, January-March, 1919.
Vols. II-III., Nos. 3-6, May-November, 1919.

There were no issues in the calendar year 1918.

Beginning with January, 1920, the size and style of the journal were changed. It was issued bi-monthly until May, 1922, and since then monthly. (There were ten issues in the calendar year 1922, Vol VI.) By cooperation with the Association of Scientific Apparatus Makers of the United States the scope of the journal has been enlarged, as indicated by the new title, "Journal of the Optical Society of America and Review of Scientific Instruments," which was adopted with the issue for January, 1922.

* Postponed from St. Louis, Dec. 1919

CONSTITUTION OF THE OPTICAL SOCIETY OF AMERICA

ARTICLE I.—NAME

The name of this society shall be THE OPTICAL SOCIETY OF AMERICA.

ARTICLE II.—AIM AND PURPOSE

It is the aim and purpose of this society to increase and diffuse the knowledge of optics, to promote the mutual interests of investigators of optical problems, of designers, manufacturers, and users of optical instruments and apparatus of all kinds and to encourage cooperation among them.

ARTICLE III.—OFFICERS

The officers of this society shall be a president, a vice-president, a treasurer, a secretary, and an executive council consisting of the above named officers of the society, the retiring president of the society and four members at large.

ARTICLE IV.—MEMBERSHIP

Membership in the society shall be either honorary, regular or associate. Associate members may be either individuals or corporations.

ARTICLE V.—AMENDMENTS

This constitution shall be amended when the proposed amendment is favored by four-fifths of all the members voting upon it. A copy of every proposed amendment shall be mailed to each regular member of the society at least thirty days before a vote is taken. Voting shall be by ballot.

Amendments to the Constitution

1. The editor of the Journal shall be, ex officio, a member of the executive council.
2. The assistant editor-in-chief and business manager of the Journal shall be, ex officio, a member of the executive council.

REVISED BY-LAWS

(The following is a transcript of the BY-LAWS *as modified by all amendments prior to Nov. 1, 1923*. The original BY-LAWS were published in Jour. Op. Soc. Am., Jan.-March, 1919, pp. 46-47. For records of amendments, see: Jour. Op. Soc. Am., VII, pp. 145-146; Jan. 1923, and Jour. Op. Soc. Am., VII, p. 211; Feb. 1923; and Jo. Op. Soc. Am. VIII, p. 38; 1924.)

I.—MEMBERSHIP

Section 1. Eligibility. Any person who has, in the opinion of the council, contributed materially to the advancement of optics shall be eligible to regular membership in the society. Any person or corporation interested in optics is eligible to associate membership. Any person who has done eminent service in the advancement of optics is eligible to honorary membership.

Section 2. Application for Membership. Applications for membership in this society will be received when stating the candidate's name and qualifications for membership in the society, and when signed by two members of the society.

Section 3. Election to Membership. All applications for membership shall first be considered as applications for associate membership and acted upon as such. Applicants shall be deemed elected to associate membership when their duly prepared applications have received the written approval of three members of the executive council.

Elections to regular membership shall be made only by the executive council in annual meeting.

Associate members may be elected to regular membership when favored with the vote of two-thirds of the members of the executive council present and voting.

The executive council may elect honorary members by unanimous vote.

Section 4. Duties and Privileges. All regular members shall have the right to vote at elections, hold office, attend meetings and receive the official publications of the society. Associate members shall have similar privileges except that they may not vote at elections nor hold office in the society. Honorary members have all the privileges of regular members but need not pay dues.

Section 5. Termination of Membership. Membership in the society may be terminated or the names of the members may be placed upon the non-resident list by vote of the council. Failure to pay dues for one year shall be deemed sufficient cause for termination of membership.

II.—Collections and Disbursements

Section 1. Dues. All regular and associate individual members of the society shall contribute five dollars annually and corporation members fifty dollars annually toward the expenses of the society. Individual members may become life members by the payment of fifty dollars. The payment of one thousand dollars or more toward the expenses of the society by an corporation or individual entitles them to the rank of patron of the society.

Section 2. Disbursements. Disbursements and assessments shall be made only when recommended by the treasurer and favored by two-thirds vote of the council. No officer or member of the society except the editor-in-chief and the assistant editor-in-chief and business manager of the Journal shall receive any remuneration for his services.

III.—Officers

Section 1. Regular Officers. The duties of the president, vice-president, treasurer and secretary of the society shall be the usual ones performed by such officers.

Section 2. Executive Council. The executive council shall direct all affairs and activities of the society not otherwise provided for by the Constitution as well as perform those duties specifically assigned to it.

Section 3. Committees. The president shall appoint, with the approval of the council, such committees as may further the objects of the society, including a committee on Publications, and one or more committees on Progress. The treasurer, the secretary and the chairmen of the various committees shall make formal reports to the society at least once each year.

IV.—Election of Officers

Section 1. Eligibility. Officers shall be elected from among the regular and honorary members of the society. The president and vice-president shall be elected for a two-year term, the secretary and treasurer for a five year term of office and shall not be eligible to re-election. Members at large of the council shall be elected for a two-year term and shall be eligible to but one re-election.

Section 2. Time of Election. All officers shall be elected in October to take office on the first of January following.

Section 3. Mode of Election. Election shall be by ballot. Nominations shall be made by a nominating committee appointed by the president and approved by the council. This committee shall consist of five regular members of the society, at least one of whom shall be a past president of the Society.

V.—Publications

This society shall publish a journal devoted to the advancement of optics. The editor-in-chief and the assistant editor-in-chief and business manager of the Journal and six or more

associate editors shall be elected by the executive council of the society. The terms of office of the editor-in-chief and the assistant editor-in-chief and business manager shall be six years, and the expiration of their respective terms shall not coincide. The terms of office of the associate editors shall be six years, one third of them to be elected biennially. Every regular, honorary and associate member of the society shall be entitled to receive the Journal.

VI.—LOCAL SECTIONS

Local sections of the society may be formed in any locality, with the advice and consent of the council, for the purpose of holding meetings and promoting cooperation. The affairs of such local sections shall be entirely in their own hands.

VII.—MEETINGS

General meetings of the society shall be held as decided upon by the executive council.

VIII.—REVISION OF THE BY-LAWS

By-Laws may be enacted, amended or suspended by concurrence of two-thirds of the whole executive council.

OFFICERS

1922-1923

PRESIDENT—Leonard T. Troland, (Term expires Dec. 31, 1923.)
Emerson Hall, Harvard University,
Cambridge, Mass.

VICE-PRESIDENT—Herbert E. Ives, (Term expires Dec. 31, 1923.)
Western Electric Company,
463 West Street,
New York, N. Y.

SECRETARY—Irwin G. Priest, (Term expires Dec. 31, 1926.)
Bureau of Staudards,
Washington, D. C.

TREASURER—Adolph Lomb, (Term expires Dec. 31, 1926.)
Bausch and Lomb Optical Company,
Rochester, N. Y.

MEMBERS OF THE EXECUTIVE COUNCIL

The above officers.

Ex officio Members:

The Past President, J. P. C. Southall, Columbia University.
The Editor of the Journal, Paul D. Foote, Bureau of Standards, Washington, D. C.
The Asst. Editor and Business Manager of the Journal, F. K. Richtmyer, Cornell University, Ithaca, N. Y.

Elected Members: (Terms expire Dec. 31, 1923).

Adelbert Ames, Jr., Dartmouth College.
H. G. Gale, University of Chicago.
Ernest Merritt, Cornell University.
W. E. Forsythe, Nela Research Laboratories.

OFFICERS

1924-1925

PRESIDENT—Herbert E. Ives, (Term expires Dec. 31, 1925.)
463 West Street,
New York City, N. Y.

VICE-PRESIDENT—W. E. Forsythe, (Term expires Dec. 31, 1925.)
Nela Research Laboratories,
Nela Park, Cleveland, Ohio.

SECRETARY—Irwin G. Priest, (Term expires Dec. 31, 1926.)
Bureau of Standards,
Washington, D. C.

TREASURER—Adolph Lomb, (Term expires Dec. 31, 1926.)
Bausch and Lomb Optical Company,
Rochester, New York.

MEMBERS OF THE EXECUTIVE COUNCIL

Ex officio Members:

The above officers.
The Past President, Leonard T. Troland, (Term expires Dec. 31, 1925.)
The Editor-in-Chief, of the Journal, Paul D. Foote. (Term expires Dec. 31, 1925.)
The Assistant Editor-in-Chief and Business Manager of the Journal, F. K. Richtmyer. (Term expires Dec. 31, 1927.)

Elected Members: (Terms expire Dec. 31, 1925.)

K. T. Compton, Palmer Physical Laboratory, Princeton, N. J.
Carl W. Keuffel, Keuffel and Esser, Hoboken, N. J.
P. G. Nutting, 4 Union Street, Schenectady, N. Y.
F. E. Wright, Carnegie Geophysical Laboratory, Washington, D. C.

PATRONS OF THE SOCIETY

(The payment of one thousand dollars or more toward the expenses of the society by any corporation or individual entitles them to the rank of patron of the society.—*By-Laws, Art. II, Sec. 1.*)

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The Measurement of Thermal Conductivity.—An accurate knowledge of the thermal conductivities of materials is important for a large number of industrial purposes, but "thermal conductivity is a physical constant not capable of ready determination by simple appliances." The present paper is the first of a series of communications on the subject and is devoted to a description of methods and technique developed for measuring conductivities of large numbers of samples of material of low conductivity. Heat is supplied to the material in the form of a thin plate, by placing it in contact with an electrically heated copper plate. Heat conducted through the specimen is received by another copper plate cooled by water circulation. The "input" heat is measured electrically; the "output" heat is measured by the change of temperature of the water stream (constant flow calorimeter method), the necessary corrections being made for end and side leakage from the specimen and the plates. With proper precautions the "input" and "output" heats were within one or two per cent of each other. There is also discussed the effect of such factors as thermal resistance at the hot and cold faces, thickness of the sample, effect of several layers of such materials as mica or fabrics, pressure on sample, etc. Three types of apparatus are described: (1) for those materials of slightly compressible nature, such as cardboard and various woods; (2) for those materials of which the contact resistance is high, so that amalgamation at the hot and cold surfaces becomes necessary; (3) for those materials available only in very thin layers.

Results are given in tabular form for a large number of woods and commercial materials such as vulcanized rubber, cotton fabric, cardboard, etc. The conductivity of rubber decreases with increase in the amount of rubber in the sample. Moisture increases the conductivity of wood. In wood, the conductivity parallel to the grain is about twice as great as in either of the mutually perpendicular directions. The radial (i.e. from center of tree) conductivity is usually some 5 to 10 per cent greater than the tangential (to the annual rings) conductivity. A precision of 1 to 2 per cent is attained, ample for commercial purposes. [E. Griffiths and G. W. C. Kaye: Proc. Roy. Soc., *104*A, pp. 71-97; 1923.]

F. K. RICHTMYER

INSTRUMENT SECTION

THERMIONIC VACUUM TUBES AND THEIR APPLICATIONS[1]

By Robert W. King

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I. Introduction

1. *Thermionic Emission.* By thermionic vacuum tubes we shall understand those whose operation depends in an essential manner upon thermionic emission.

The design of the various types of thermionic tubes at present in use requires no knowledge of the exact mechanism of thermionic emission. It may be said, however, that the work of O. W. Richardson and others leaves little question but that this emission is a physical as distinguished from a chemical process, and occurs from certain substances as the result of the large velocities of thermal agitation acquired by electrons when these substances are raised to a high temperature.

[1] The present material was originally prepared for the National Research Council for use in a proposed *Manual* on "Physical Research Methods and Technique." As the appearance of the *Manual* has been postponed, the Committee in charge of its preparation has kindly consented to the separate publication of some of the sections in various technical magazines. In order to meet the requirements of the *Manual*, the form of expression has been made as compact as possible with practically no discussion of theory and no derivation of formulas. Since this syle of presentation leaves much to be desired from some points of view, references have been given to the original literature wherever possible. However, many of the vacuum tube circuits presented here have not as yet been treated in the literature. In the preparation of the new material the author has been greatly helped by persons whose contact with these subjects is at first hand.

On the basis of certain plausible assumptions, O. W. Richardson derived[2] the expression,

$$I_s = Ne = AT^{\lambda}\epsilon^{-\frac{w_0}{Tk}}, \tag{1}$$

for the thermionic emission per cm² in which I_s is the saturation current formed by drawing all the emitted electrons to a positively charged electrode placed near the emitting surface, e is the electronic charge and ϵ the Naperian base, A is a constant dependent on the emitting substance but independent of the absolute temperature T, w_0 represents the energy lost by each electron as a result of becoming free, λ is a number which does not differ much from unity, and k is the gas constant per molecule. Experiments show that the value of λ for a wide range of substances is about unity, but its exact value is of little practical importance, since the variation of I_s with T is almost entirely controlled by the term in which T enters as an exponent.

The quantity w_0 which expresses the *electron affinity* of the emitting substance is usually called the internal work of evaporation. In Equation 1, it is in terms of ergs per electron. Calling v_0 the value of w_0 when expressed in equivalent volts, $w_0 = 1.59 \times 10^{-12} v_0$.

The term v_0 is of great importance when considering the economy with which a substance acts as a thermionic emitter. Assuming that the emission of an electron occurs when its velocity acquires a value sufficiently high to overcome the potential drop v_0, it is apparent that the smaller v_0, the more copious will be the thermionic emission at any given temperature. For the substances thus far examined, v_0 ranges between about two volts and five volts.[3] The two substances whose thermionic properties we shall consider particularly are platinum, coated with a mixture of barium and strontium oxides, and tungsten. For tungsten the value of v_0 is approximately 5 volts, and for coated platinum it varies between 1.67 and 2.05 volts. The value of $(v_0)_A - (v_0)_B$ for two substances A and B is equal (except for a small term expressing the Peltier coefficient) to their contact difference of potential.[4]

2. *Thermionic Properties of Filaments.* In designing electron tubes with predetermined characteristics knowledge of the thermionic emissivity of the proposed filament is necessary. This property may

[2] Richardson, The Electron Theory of Matter, 1916 Edition, page 441.

[3] For a Table of Values of v_0 for the materials commonly used see Van der Bijl—Thermionic Vacuum Tubes, page 29; also Langmuir, Trans. Am. Electrochem. Soc. *29*, 166, 1916.

[4] Richardson, Electron Theory of Matter, 1916, p. 455.

be conveniently represented by curves of the type shown in Figs. 1, 2 and 3. The coordinates have been so proportioned[5] that, provided the electronic emission varies with the temperature as indicated by

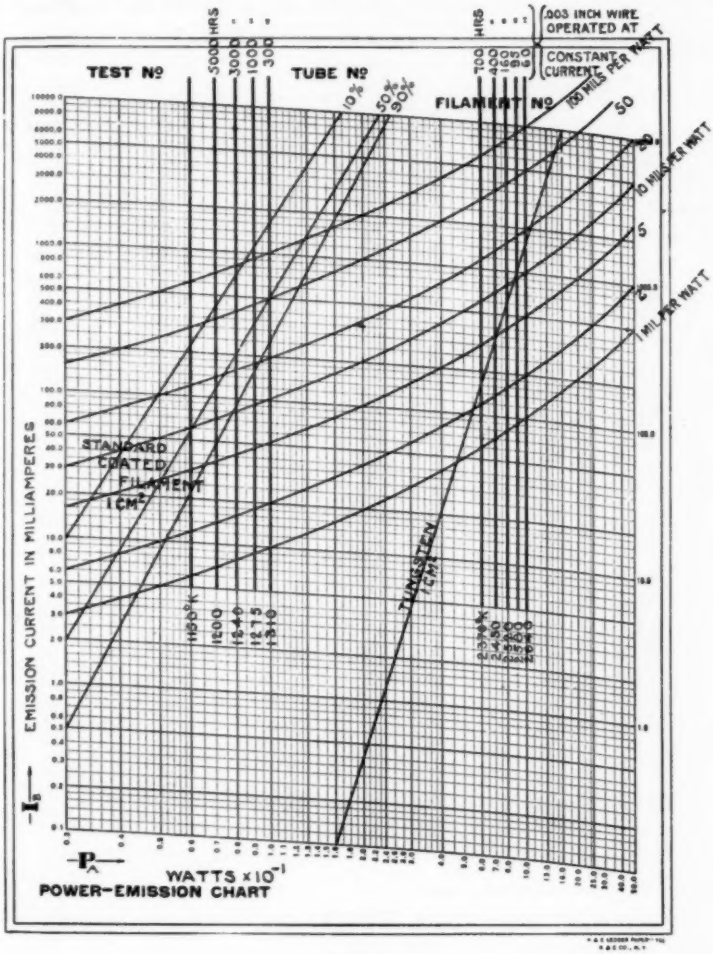

FIG. 1

Equation 1 and the thermal radiation from the filament varies as the fourth power of the temperature, then the relation between the ther-

[5] See Van der Bijl, The Thermionic Vacuum Tube, p. 82.

mionic emission and the heating power supplied to the filament is a straight line.

Fig. 1 gives data for tungsten and for coated platinum filaments, Fig. 2 compares thoriated tungsten filament with pure tungsten and Fig. 3 gives data relating to a special coated filament, the core of which

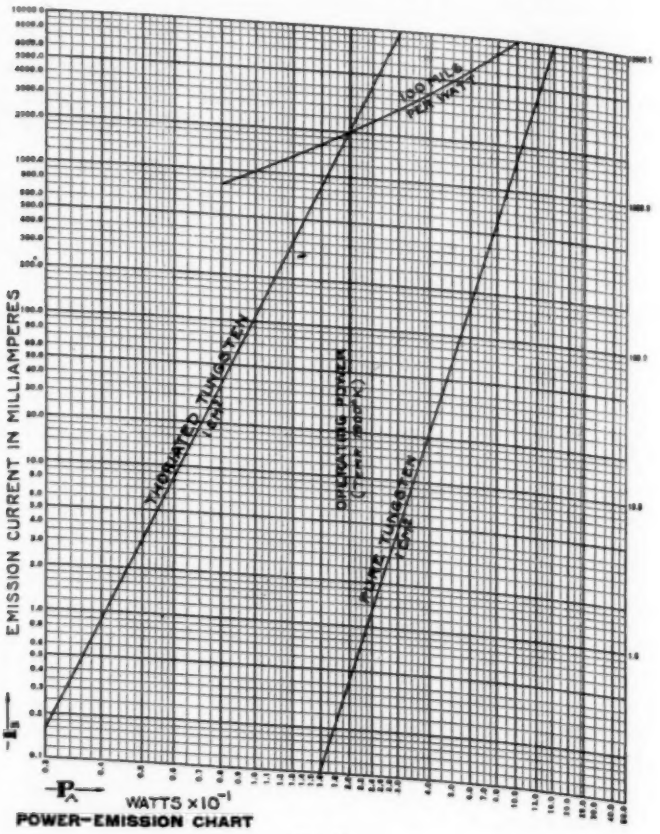

FIG. 2

consists mainly of platinum alloyed with about 5% of nickel. Since the thermionic emission of tungsten, and thoriated tungsten when freshly activated, do not vary much between different samples, they are given with sufficient accuracy for tube design purposes by single lines. Coated filaments, however, show a rather wide variation as is

indicated. The range of variation shown in Fig. 1 was obtained in the study of severa thousand tubes; 10 per cent. showed activity greater than that represented by the 10% line, 50 per cent. showed greater activity than the 50% line and 90 per cent. showed activity

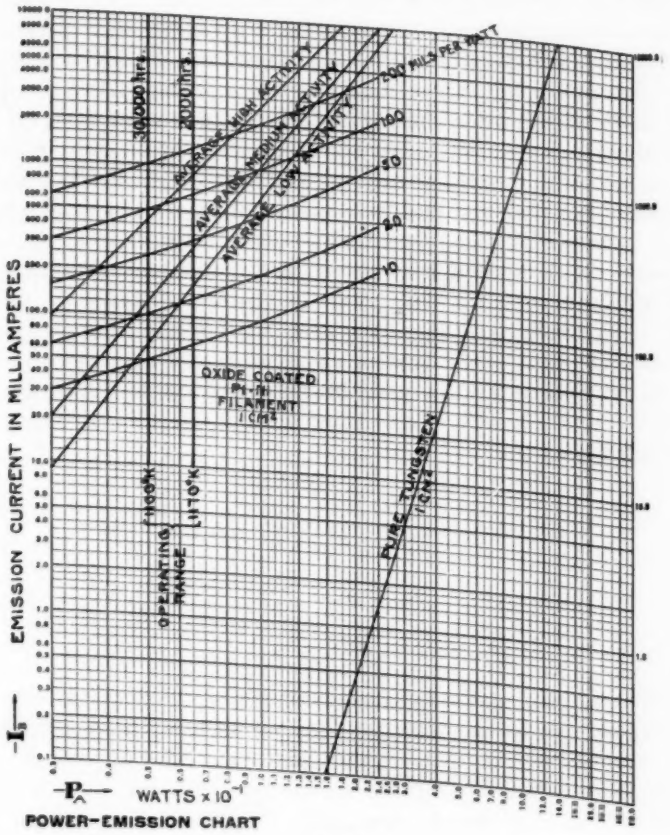

FIG. 3

greater than the 90% line. The three lines illustrating the range of variation in Fig. 3 do not correspond to these same percentages but have been labeled so as to be readily interpretable.

The efficiencies in mill'amperes of emission current per watt used to heat the filament are shown by the curved lines that cross each chart. Operating temperatures and corresponding filament life are given for

certain of the ordinates. The tungsten life data in Fig. 1 are for 3-mil wire and a constant heating current. If operating at a constant temperature, the life would be somewhat longer; furthermore, the larger the wire the longer the life for a given temperature. The activity of thoriated filaments tends to diminish with use but may be re-activated by heating to a temperature higher than the normal operating temperature. The useful life of these filaments is, therefore, somewhat indefinite but taking the possibility of re-activation into account, it is probably well in excess of 2,000 hours.

In using Figs. 1, 2, 3 it should be borne in mind that the emission values given represent saturation currents and that in general the normal operating space current in a tube must, for reasons which will appear later, be appreciably less than the saturation current. The difference between the saturation current and the maximum operating space current varies with the duty to which the tube is assigned. In the case of high voltage rectifiers, the space current may at certain points in the cycle reach the saturation value, while in a tube which is used as an amplifier it is often desirable, in order to avoid distortion, to have the total emission two to three times as great as the maximum working space current.

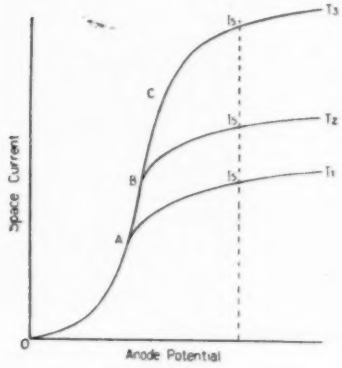

FIG. 4

3. *Space Current-Voltage Characteristic.* Experiment shows that in a vacuum tube containing an emitting electrode and a conveniently placed anode, the space current I_p, varies with the temperature of the emitter and the anode potential E_p, as in Fig. 4. The three curves shown are for three temperatures such that $T_1 < T_2 < T_3$. It will be observed that between points O and A the three curves coincide;

between O and B the curves for the two higher temperatures coincide. The saturation values of the filament emission at the various temperatures are shown by I_s.

For values of I_p ranging from zero to somewhat less than I_s, the relation between I_p and E_p may be expressed with a fair degree of accuracy by $I_p = \kappa E_p^{\eta}$ in which the exponent η does not differ greatly from 3/2. This relation, therefore, is frequently known as the 3/2 power law. It has been deduced theoretically by Child[6] and Langmuir[7] and has been studied lately in greater detail by Fry.[8] Fry's analysis takes account of the initial velocities of emission of electrons, and, as he shows, the effect of the space charge is to create a region of negative potential immediately around the emitter. Let Fig. 5

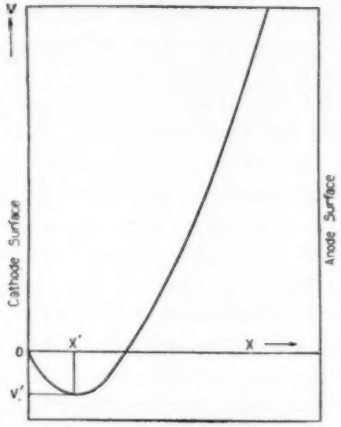

FIG. 5

represent the value of the potential as one proceeds from the cathode in the direction x, and V' represents the minimum value of the potential. Assuming indefinitely large emission from the cathode, V' (which is a function of E_p), determines the space current corresponding to any particular value of E_p. The lower the value of V' the fewer the electrons with initial velocities sufficient to carry them past the equipotential surface V' into the region where they are attracted by the anode. Assuming the average initial velocity to be 0.3 volts (roughly a temperature of 2400°K), Fry finds an appreciable deviation from the 3/2 power law for $E_p < 30$ volts, but initial velocities need be considered only in tubes which operate at low E_p.

[6] Phys. Rev., *32*, p. 498; 1911.
[7] Phys. Rev. (2), *2*, p. 450; 1913.
[8] T. C. Fry, Phys. Rev., *17*, p. 441; 1921.

Another factor which, for low E_p causes an appreciable deviation from the 3/2 power law, is the potential gradient in a filament cathode due to the heating current. Whereas velocity of emission tends to make $\eta < 3/2$, the potential gradient in the filament has the reverse effect. In general, the latter more than overbalances the former and for small anode voltages $\eta < 3/2$. The value of η when the cathode potential gradient is considered, but initial velocities are neglected, has been given by W. Wilson,[9] who finds that when E_p is less than the potential drop across the filament $\eta = 5/2$, while for higher E_p it gradually approaches the limiting value 3/2.

4. *Temperature Saturation and Voltage Saturation.* When a vacuum tube operates at such filament temperature and E_p that an increase in temperature produces no increase in I_p the tube is said to show *temperature saturation.* On the other hand when temperature and E_p are such that an increase in E_p does not increase I_p, the tube shows *voltage saturation.* Referring to Fig. 2 the curve T_1 shows temperature saturation between O and A and approaches voltage saturation beyond the point A. Similarly the curve T_2 shows temperature saturation up to the point B and approaches voltage saturation beyond.

5. *Effect of Gas.* In thermionic tubes as usually pumped the gas pressure is between 10^{-5} and 10^{-6} mm. At this pressure the gas generally does not manifest its presence in the operation of the tube. However, at higher pressures, and particularly above 10^{-4} mm, it produces certain objectionable disturbances. Thus many gases seriously reduce the filament activity; also for E_p greater than about 20 volts, ionization occurs and the resulting discharge differs in important respects[10] from the pure electron discharge of Fig. 4.

II. TWO-ELECTRODE TUBES

The two-electrode tube, which was first due to Edison, found an early practical application when Fleming used it to detect wireless telegraph signals.

However, since the advent of the three-electrode tube of De Forest, the earlier device has been almost entirely superseded as a detector and finds its principal application as a rectifier of a.c. voltages. Its range of applicability in this field is extremely large. With properly designed tubes, Hull[11] has succeeded in rectifying 5 k.w. at a potential

[9] For discussion of the 5/2 power relation, see Van der Bijl, The Thermionic Vacuum Tube, p. 64.

[10] See Van der Bijl: The Thermionic Vacuum Tube, p. 86.

[11] A. W. Hull, General Electric Review, *19*, p. 177, 1916. Another good source ofinformation is Van der Bijl's "The Thermionic Vacuum Tube."

of 100,000 volts, and in its transatlantic radio telephone experiments, the American Telephone and Telegraph Company is using a six phase rectifier giving about 200 k.w. at about 10,000 volts.[12]

6. *Two-Electrode Tube as Rectifier.* Three typical forms of circuit are shown in Figs. 6, 7, 8, each of which has certain characteristics

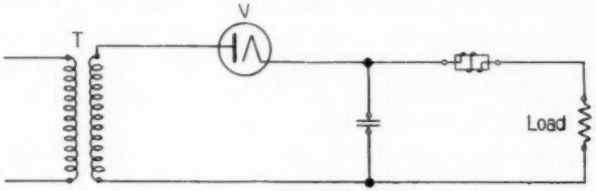

FIG. 6

not possessed by the others. The circuit shown in Fig. 6 rectifies the full transformer voltage, but utilizes only one-half of the current wave; the circuit of Fig. 7 gives a d.c. voltage of only about half the

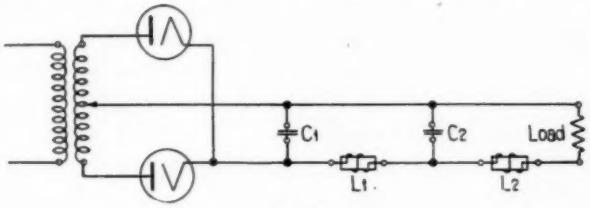

FIG. 7

transformer peak voltage but utilizes both halves of the wave, and Fig. 8 illustrates a circuit making use of the full transformer voltage and both halves of the a.c. wave.

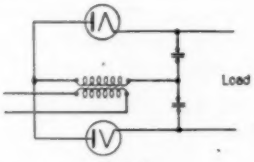

FIG. 8

A rectifier circuit employing a tuning condenser for the secondary of the high voltage transformer and giving a rectified d.c. voltage as large again as the a.c. voltage of the transformer and providing auto-

[12] See Arnold and Espenschied, Journal of A. I. E. E., August, 1923; also Bell System Technical Journal, October, 1923.

matic control of the maximum d.c. voltage supplied by the rectifier, has been described by Webster.[13]

The circuits shown in Figs. 6 and 7 provide means in the form of condensers C_1, C_2 and inductances L_1, L_2 for smoothing out the rectified voltage. Such an arrangement of conductors is essentially a network whose attenuation for electric currents of a certain range of frequencies is very low and for all other frequencies is high. This type of network is a special form of the *electric wave-filter* which is finding many applications at the present time and a complete account of its properties is to be found in recent literature.[14]

No definite statement can be made as to the exact range of frequencies over which the rectification of alternating currents can be satisfactorily accomplished by means of thermionic tubes, but it is apparent that this range is large. The degree of smoothness required in the d.c. output is of primary importance in setting the lower limit of the frequency range; on the other hand, the smaller the load resistance, the higher the frequency which may be satisfactorily rectified before the internal capacity of the tubes permits the flow of an objectionable amount of alternating current. Whenever an output with a minimum of ripple is required it is in general desirable to use as high a frequency as is readily available.

III. THREE-ELECTRODE TUBES

In 1906 De Forest brought out the three-electrode tube[15] in which a grid is interposed between filament and plate. Since the introduction of this tube, much study has been devoted to its properties and many investigations have been made concerning the best substances to use as thermionic emitters, the best metals for plates and grids, the best varieties of glass for the containing bulbs,[16] and the best methods of exhaustion,[17] so that today problems of design are well understood. At present the three-electrode tube finds use as a rec-

[13] D. L. Webster, Procd. Nat. Acad., *6*, p. 28, p. 269: 1920. These articles also suggest certain modifications of Hull's method.

[14] G. A. Campbell, Bell System Technical Journal, Nov., 1922; U. S. Patents 1,237,113 and 1,237,114, May 22, 1917; O. J. Zobel, Bell System Technical Journal, Jan., 1923; Carson and Zobel, Bell System Technical Journal, July, 1923; G. W. Pierce, Electric Oscillations and Electric Waves, p. 186, 1920; Karl W. Wagner, Arch. f. Electr., *3*, p. 315; 1915; *8*, p. 61; 1919.

[15] Variously called the audion, vacuum tube, triode, pliotron, etc.

[16] Measurements of Gases Evolved from Glasses of Known Chemical Composition—Harris & Schumacher, Jour. Ind. & Eng. Chem., Feb., 1923; also Bell System Technical Journal, Jan., 1923.

[17] For methods of exhaustion, see Dushman, Gen. Elect. Review, *23*, p. 493; 1920.

tifier, amplifier of small currents and voltages, detector of small a.c. voltages, modulator of alternating currents, and generator of electric oscillations. Tubes have been built which range in size from those about one inch long with a space current of about a milliampere to others which are water-cooled and have an individual output capacity of 100 k.w.[18] Amplifiers with a capacity of 150 k.w. are in successful operation (see footnote 12).

7. *Action of the Grid.* The general theory of the grid action is simple. As pointed out by Fry[19] the space charge creates a region of negative potential immediately around the cathode which persists for all values of E_p less than that required to produce voltage saturation. It is the minimum potential V' (see Fig. 5) that limits I_p, and any increase in V' will result in an increase in I_p. Because the grid is close to the filament, small changes in the grid potential, E_g, are as effective in changing V' and therefore I_p, as large changes in E_p. This leads to the so-called amplification constant μ of the tube which may be taken as

$$\mu = \frac{\pm e_p}{\mp e_g},$$

in which $\pm e_p$ and $\mp e_g$ are changes in E_p and E_g, the changes being opposite in sign as indicated, and such that they leave I_p unchanged.

It has also been shown[20] that if ΔE_p and ΔE_g are increments of E_p and E_g which cause equal increments in the electric field at the surface of the cathode (considered simply as an equipotential surface and not as a source of electrons) the amplification constant, μ, of the tube will be the ratio $\Delta E_p/\Delta E_g$.

8. *Characteristic Equation.* The electrical characteristics of the three-electrode vacuum tube may be represented[21] by the equation

$$I_p = \kappa\left(\frac{E_p}{\mu} + E_g + \epsilon\right)^{\eta}. \qquad (2)$$

The constant κ is related in a simple way to the internal resistance of the tube. A consideration of ϵ which expresses the contact difference of potential between grid and filament is usually essential only in tubes which operate at a low E_p and particularly in detectors and amplifiers. In tubes with coated filament, ϵ may not only vary

[18] See W. Wilson, Bell System Technical Journal, July, 1922.
[19] T. C. Fry, l. c.
[20] R. W. King, Phys. Rev., *15*, p. 256; 1920.
[21] Van der Bijl, Phys. Rev., *12*, p. 180; 1918.

within a range of two or three volts between different tubes, but may also change during the life of any one tube. The exponent η varies in a given tube with applied voltage, being usually equal to about 2 when the effective voltage $\left(\frac{E_p}{\mu}+E_g+\epsilon\right)$ is comparable with the potential drop in the filament (see Fig. 9), and tending to approach the theoretical value 3/2 with increasing effective voltage. It has been

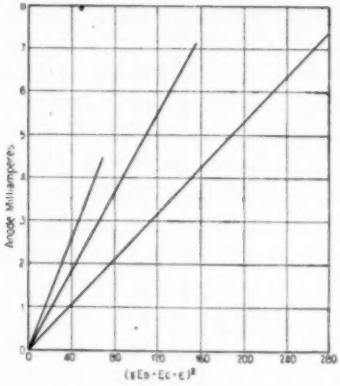

FIG. 9

found possible to deduce relations between the constants μ and k of Equation 2 and the structure and dimensions of any tube which are in very fair agreement with experimental values.[22]

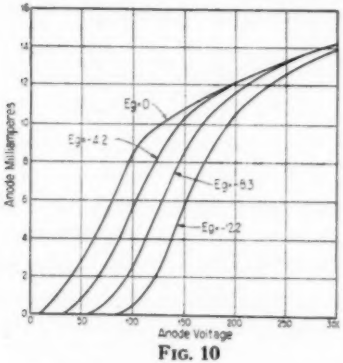

FIG. 10

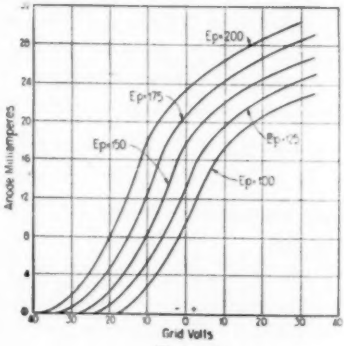

FIG. 11

Typical curves corresponding to the characteristic Equation 2 are shown in Figs. 10 and 11. These curves are referred to as *static* char-

[22] King, l. c.

acteristics one parameter being fixed in each case. For the *dynamic* characteristic see section 12. Equation 2, of course, fits only the portions of the curves characterized by temperature saturation.

The abruptness with which the curves of Fig. 10 meet the potential axis is important in certain uses to which tubes are put. The value of E_g which reduces I_p to zero is called the *cutoff voltage*. To have a sharp cutoff a tube should have a fairly large μ and its grid should be sufficiently large with respect to the filament to effectively screen all parts of the filament from the plate.

9. *Grid Current.* For certain purposes, a consideration of the grid current I_g, is necessary. Fig. 12 represents a characteristic relation between I_g and E_g for various E_p. Note that E_g in excess of about

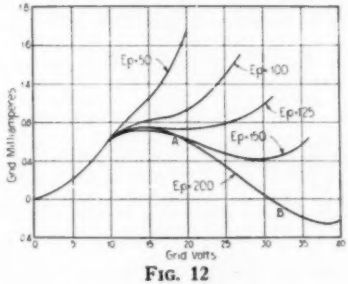

FIG. 12

10 volts results in *secondary emission* of electrons from the grid. These secondary electrons flow to the plate and, as shown, their number may actually exceed the total number of primary electrons striking the grid. The character of the grid surface plays an important part in determining the amount of secondary emission. The secondary emission from the grid of a tube containing a pure tungsten filament is, in general, less than that from the grid of a tube with an oxide-coated filament. At high temperature a coated filament appears to evaporate a minute amount of its coating, some of which is deposited upon the grid presumably augmenting the secondary activity.[23]

10. *Vacuum Tube Constants.* The two most important constants of the three electrode tube are μ and its internal resistance r_p. The determination of μ and r_p from the characteristic curves (Figs. 10, 11) is obvious. For general design purposes these curves give the best

[23] A. W. Hull has designed two types of tube known as the dynatron and pliodynatron which utilize the negative resistance characteristic (AB of Fig. 11) resulting from secondary emission. Procd. Inst. Radio Engrs., *6*, p. 5; 1918.

insight into the behavior of a tube and furnish the most instructive means of determining μ and r_p.

11. *Dynamic Methods of Measuring Vacuum Tube Constants.* However, in cases where many tubes, all practically alike, have to be tested, certain "dynamic" methods are timesavers. Several such methods have been devised, but all are modifications of a scheme first published by Miller.[24]

Miller's method is illustrated in Fig. 13. To determine μ the key K_1 is open and K_2 is closed. The resistance r_1 is adjusted until the

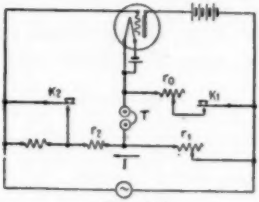

FIG. 13

sound heard in the telephones T is a minimum, under which circumstances it is clear that $\mu=\frac{r_1}{r_2}$. To determine r_p, some definite relation between r_1 and r_2 is established. Then, with key K_1 closed, the resistance r_0 is adjusted until the telephone response is a minimum. With this adjustment it may be shown that

$$r_p=r_0\left(\mu\frac{r_2}{r_1}-1\right). \tag{3}$$

This measurement of r_p may be simplified as follows: suppose we adjust r_1 for a minimum tone in T when K_1 is open. Then $\mu=\frac{r_1}{r_2}$, and it is seen from Equation 3 that with this relation between r_1 and r_2 it would not be possible to obtain a balance with K_1 closed; but if r_2 be doubled, which can be done by opening K_2 thus adding a resistance equal to r_2, and r_0 be now adjusted with K_1 closed to give a minimum tone in T, then $r_p=r_0$.

12. *Dynamic Characteristics of Vacuum Tubes.* In a circuit such as that shown in Fig. 13, the space current causes a fall of potential along any resistance r, and the difference in potential between filament and plate is therefore less than the potential difference across the battery by the amount $I_p r$. If I_p is increased in any way, as for

[24] J. M. Miller, Proc. I. R. E., 6, p. 141; 1918. For variations of Miller's dynamic method the reader is referred to Van der Bijl, l. c., p. 198, Method of G. H. Stevenson.

instance, by an increase in E_g, the drop $I_p r$ increases and with a fixed battery e.m.f. the potential difference between the filament and plate diminishes somewhat. It follows, therefore, that a given change in E_g will cause a smaller change in space current when the plate circuit includes an external resistance r than when it does not.

This important fact supplies a simple means of straightening the characteristic of a vacuum tube to such an extent that it may become practically a distortionless amplifier.

To a first approximation,[25] the alternating component J of the space current, when a voltage $e = e_0 \cos pt$ is applied to the grid is given by

$$J = \frac{\mu}{r + r_p} e_0 \cos pt - \frac{r_p r_p' e_0^2}{2!(r + r_p)^3} (1 + \cos 2pt). \qquad (4)$$

In this equation r_p is the internal resistance of the tube and r'_p is its derivative with respect to the effective voltage $\left(\frac{E_p}{\mu} + E_g\right)$. It will be noted that the second term on the right side of Equation 4, which gives the first harmonic, diminishes rapidly with r as was to be expected from the preceding paragraph. The more or less straightened

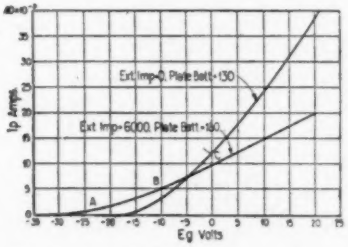

FIG. 14. *Average I_p, E_g Characteristics for Five Tubes. Average $\mu = 5.92$; average internal impedance = 6000 ohms for $E_b = 130$ volts and $E_g = -9$ volts. Plate battery connected to + end of filament and grid battery to − end. Western Electric Type 101–B tubes.*

characteristic resulting from the effect of r is known as the *dynamic* characteristic; see Fig. 14, which curves fit a tube whose $r = 6{,}000$ ohms. As will be pointed out in the section on amplifier circuits, the dynamic characteristic is a useful guide in selecting tubes as amplifiers.

Equation 4 also expresses the important fact that the application of the voltage e_0 to the grid is equivalent, so far as current in the plate

[25] J. R. Carson, Proc. Inst. Radio Engrs., 7, page, 187; 1919. In case the output circuit of a tube contains reactance as well as resistance, Eq. 4 becomes much more complicated as Carson shows.

circuit is concerned, to the application of the voltage μe_0 in the plate circuit.

13. *Internal Capacities and Effect on Input Impedances.* In certain uses to which the vacuum tube may be put, a knowledge of the influence which the internal electrostatic capacities have on the input impedance is important. The equivalent circuit of the tube[26] is shown in Fig. 15 in which C_1 is the capacity between filament and grid, C_2 capacity between filament and plate, C_3 capacity between grid and

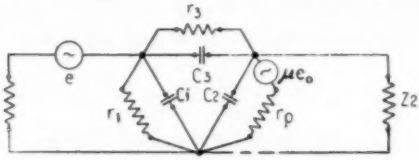

FIG. 15

plate, and r_1, r_3, are leakage resistances. As the action of the tube is such as to produce an equivalent voltage μe_0 between filament and plate, a generator of voltage μe_0 is shown in series with the internal resistance r_p of the tube. Calling Y_g the input admittance of the tube, that is,

$$\frac{1}{Y_g}=Z_g=\frac{e_g}{i_1+i_3},$$

in which e_g is the alternating voltage between filament and grid, the solution of the above circuit gives for Y_g the value,

$$Y_g=\frac{1}{r_1}+jC_1p+\frac{\left(\frac{1}{r_3}+jC_3p\right)[jC_2pr_pZ_2+r_p+Z_2(\mu+1)]}{(jC_2pr_pZ_2+r_p+Z_2)+\left(\frac{1}{r_3}+jC_3p\right)r_pZ_2}. \tag{5}$$

Case 1, *Low Frequencies.* For low frequencies the admittance of the condenser C_2 is negligible compared with that of r_p. Dropping the terms containing C_2 gives the equation,

$$Y_g=\frac{1}{r_1}+jC_1p+\left(\frac{1}{r_3}+jC_3p\right)\frac{r_p+Z_2(\mu+1)}{(r_p+Z_2)+r_pZ_2\left(\frac{1}{r_3}+jC_3p\right)},$$

which yields important interpretations. In case the load impedance Z_2 is a pure resistance r_2, the admittance of the filament-grid branch

[26] H. W. Nichols, Phys. Rev., *13*, p. 405; 1919; J. M. Miller, Bureau of Standards, Bulletin No. 351.

of the tube may be much greater than the admittance which would result from R_1 and C_1 alone. This is due to the influence which the alternating component of the plate voltage exerts upon the input circuit through the condenser C_2. Figs. 16 and 17 show respectively the effective capacity and effective conductance between filament and grid as a function of the external resistance. For the particular tube studied (W. E. Co. 102-A) Fig. 17 shows that, if $r_2 = 40{,}000$ ohms, the effective capacity between filament and grid is not the capacity C_1 (about 10×10^{-12} farads) but is approximately 120×10^{-12} farads. Fig. 16 shows that the effective conductance is also greatly increased above that due to r_1. This increase in conductance means an increased absorption of input energy by the tube which, of course, is not dissipated in the grid circuit but passes through the path supplied by the condenser C_2 to be wasted in the plate circuit.

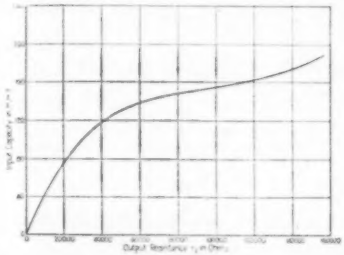

FIG. 16

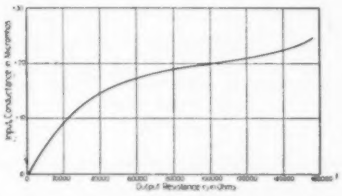

FIG. 17

In case Z_2 is a pure inductance L_2, the effective input conductance of the tube is negative and not positive as in the preceding case. Fig. 18 shows the variation of this negative conductance with L_2, and Fig. 19 the variation of effective input capacity with L_2. A negative input conductance means that the grid circuit draws power from the plate circuit; if the negative conductance is large enough, a tube in such a circuit will oscillate steadily or "sing" with no coupling but that provided by its internal capacities. This phenomenon is frequently encountered in vacuum tube amplifiers and at times proves quite troublesome.

Tubes can readily be constructed in which r_1 and r_3 are so large as to exert no influence on the behavior of the tube and may be ignored in the above equations. However, even in such tubes there is an effective input conductance, either positive or negative, depending upon the character[27] of Z_2.

[27] For cases in which Z^2 is neither pure resistance nor reactance, see Van der Bijl, l. c., p. 210.

Case 2, High Frequencies. For very high frequencies terms of the first order in p are negligible compared to terms of the second order, and Eq. 5 becomes,

$$Y_g = \frac{p(C_1C_2+C_1C_3+C_2C_3)}{C_2+C_3},$$

indicating that as the frequency is raised the effective input impedance

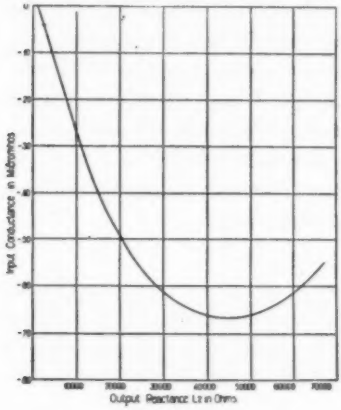

FIG. 18

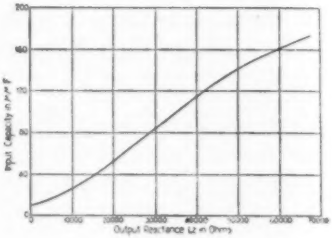

FIG. 19

approaches that due to the condensers alone. Under these circumstances the grid absorbs very little power, but the amplification is lowered because the input is to an extent short-circuited by the

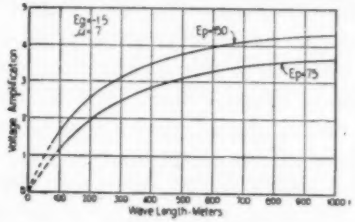

FIG. 20

electrode capacities. Fig. 20 shows the variation in voltage amplification against wave lengths in meters for high frequencies. The two curves are for different E_p, the higher E_p giving a larger amplification because r_p of the tube is lower. It is seen that the amplification at 1,000 meters is about three times as large as the amplification at 100 meters and the amplification at both values of E_p tends to approach zero as the frequency becomes infinite.

Nichols[28] suggests that the reduction in amplification for a given frequency can be avoided by shunting the grid-plate capacity C_3 with an inductance of such a value as to make the impedance between grid and plate infinite at this frequency.

IV. THERMIONIC AMPLIFIERS

Equation 4, given in Sec. 12, is of fundamental importance in the design of vacuum tube amplifier circuits. Neglecting the second term on the right hand side which, as previously pointed out, expresses distortional effects and should therefore be very small in amplifier circuits, the equation can be written

$$J = \frac{\mu e_0}{Z + r_p}, \tag{6}$$

in which the impedance $Z = r + jx$ is substituted for r. From Equation 6 both the voltage and power amplification of a tube for any particular circuit can readily be calculated.

14. *Voltage Amplification.* Assuming as above that the tube works into an output impedance Z, it follows that the voltage amplification (i.e., the ratio of the output to the input voltage) is

$$\frac{\mu Z}{Z + r_p}.$$

This expression shows that the voltage amplification increases as Z increases. Considering separately the two cases in which Z is a

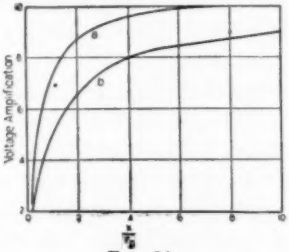

FIG. 21

pure resistance and pure reactance, typical values of the voltage amplification are plotted in Fig. 21. Curve *a* corresponds to reactance in the output and *b* to resistance. These curves show that the voltage amplification rises much more rapidly when reactance is used, reaching 90% of its maximum value when $\frac{x}{r_p} = 2$.

[28] H. W. Nichols, Phys. Rev. Vol. 13, p. 411; 1919.

If the resistance component of Z is made as small as possible, E_p becomes practically equal to the potential across the plate battery, making possible a given voltage amplification with smaller plate battery than could be obtained if the output circuit contained an appreciable resistance.

15. *Power Amplification.* From Equation 6, which neglects harmonic terms, it is readily seen that the power output is

$$\frac{\mu^2 e_0^2 r}{(r+r_p)^2+x^2},$$

where $r+jx$ has been substituted for Z. This output is a maximum when $r^2=r_p^2+x^2$. The case in which $x=0$ is particularly important; evidently for maximum power output, a tube should work into a resistance equal to its internal resistance.

As pointed out in Sec. 13 the input impedance of a tube is not always readily determinable; however, calling the input resistance[29] r_g, the power amplification produced by a tube is given by the expression

$$\frac{\mu^2 r r_g}{(r+r_p)^2+x^2}. \tag{7}$$

This expression has been obtained on the assumption that the grid draws no electron current from the space charge, which in turn requires that the grid remain at a negative potential at all times. Since the power amplification falls rapidly as the grid becomes positive, it is customary in most amplifier circuits to supply means of maintaining the grid at a negative potential.

Expressions 6 and 7 are of fundamental importance in the design of amplifier circuits.

16. *Selection of Tubes.* When selecting tubes for an amplifier, curves such as those shown in Fig. 14 are very useful. By their means it is readily possible to select the tube, the plate potential and the average grid potential which will give satisfactory results for any pre-assigned value of the input voltage. In order to obtain amplification as free from distortion as possible, it is necessary that the grid potential in its excursions neither become positive nor strike the lower end of the characteristic. To a sufficient approximation

[29] Where many tubes of the same design are to be interchanged in a given circuit, and where the conditions of manufacture are such that the insulation resistance between filament and grid is not always of the best, it may be found desirable to shunt the input with a fixed resistance, e.g., ½ megohm.

it is evident that when the variable grid or input voltage e_0 is given, we should choose E_g and E_p such that

$$e_0 \gg -E_g \gg \frac{E_p}{2\mu}.$$

Referring to Fig. 14, for an input potential of 10 volts (peak value) and $r_p = 6{,}000$ ohms, the point B (but neither A nor C) is evidently a satisfactory mean position about which to operate.

Since both voltage and power amplification increase as E_p is increased (because r_p is decreased) it is frequently desirable to have the value of E_p considerably larger than the limit just indicated.

17. *Amplifier Circuits.* The fundamentals of thermionic amplifier circuits may be gathered from Fig. 22. The variable input voltage e_0 between grid and filament of the first tube A modulates the plate current of this tube. The circuit is evidently such that the variable

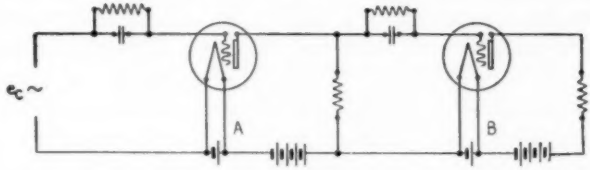

FIG. 22

I_p of tube A varies the grid potential of B and, due to the properties of the three electrode tube, not only is the power applied to the grid of B greater than that applied to A, but the potential variation may be many times as large as e_0. Hence the variations in I_p of B will be larger than those of I_p in A, thus yielding an amplifying action. Tubes A and B and their associated circuits are known as the first and second stage respectively.

Amplifier circuits may for convenience be divided into six general classes:

1. Resistance coupled circuits (Fig. 23).
2. Resistance-condenser coupled circuits (Fig. 24).
3. Retard-condenser coupled circuits (Fig. 25).
4. Transformer coupled circuits (Fig. 26).
5. Feed-back circuits (Fig. 31).
6. Push-pull circuits (Fig. 32).

The sections immediately following will point out the advantages of each type of circuit and general design considerations. Equations 6 and 7 show that the amplification of which a single tube (or "stage")

is capable is definitely limited. For greater amplification than a single stage can produce, it is necessary to arrange two or more stages in cascade. Multistage amplifiers frequently consist of combinations of certain of the above types of circuits as will be pointed out in the following paragraphs.

18. *Resistance Coupled Amplifier.* This simplest of all amplifier circuits (Fig. 23) is particularly useful where a wide range of frequencies is to be amplified without selective amplification of any particular frequencies. For this reason, it is often used as an amplifier in connection with an oscillograph. It is also one of the few types of circuit which can be used for direct current amplification. However, as will be pointed out later, a special type of push-pull circuit makes a more satisfactory d.c. amplifier for many purposes.

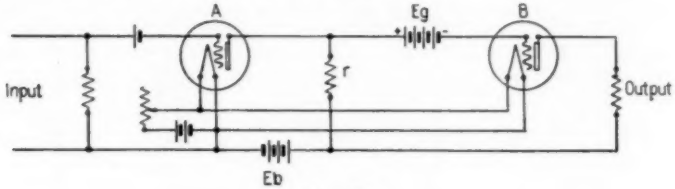

FIG. 23

One or more stages of the resistance coupled circuit may be substituted for transformers in voltage amplification. Voltage amplifier tubes having an amplification constant $\mu = 30$ are common and it follows from Equation 6 that such a tube can readily produce a voltage amplification of from 20 to 25. It can, therefore, take the place of an input transformer in one of the other types of amplifier circuit. Unless special considerations require another adjustment, it is customary to arrange all but the last stage of an amplifier for voltage amplification, the last stage being designed for power amplification. (See Secs. 14 and 15.)

Since in resistance coupled amplifiers there is a d.c. path between the plate of one tube and the grid of the following tube, a negative grid battery large enough to counterbalance the plate battery must be used in every stage in order to supply the necessary negative grid potential. As shown in Fig. 23, a common plate battery can be used for two or more stages. A more complete discussion of battery requirements is given under Power Supply.

19. *Resistance-Condenser Coupled Amplifier.* This type of circuit (Fig. 24) is similar to the preceding in all respects except that con-

densers are inserted between the plates and grids of adjacent stages. This makes the employment of large negative grid batteries unnecessary although it is still important that steady negative potentials be applied to the grid of each tube sufficiently large to prevent their being carried positive by the variations. For example, in Fig. 24, r'_g may be two meghoms and the grid battery emf 2 to 3 volts. This

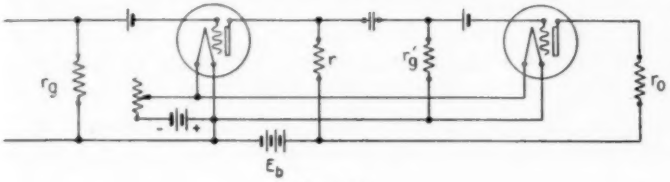

FIG. 24

type of circuit is in general the easiest to handle. Due to the insertion of condensers it will, of course, not serve as a direct current amplifier, but with sufficiently large condensers it can handle low frequencies with little or no distortion.

20. *Retard-Condenser Coupled Circuit.* The substitution of retard coils for resistances (see Fig. 25) in the circuit last described affects the behavior of the circuit in several ways. An advantage in the change lies in the fact that a given plate potential can be secured by

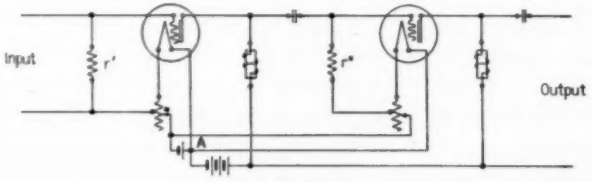

FIG. 25

a smaller plate battery (Sec. 14). Since the tubes in all but the last stage of an amplifier generally act as voltage amplifiers, it is desirable that the inductance of the retard coils be made large. By employing retard coils of the proper inductance and resistance and shunting them with condensers, such an amplifier may be tuned for any particular frequency.

It follows that the width of the frequency band which can be amplified without distortion is likely to be less than for the resistance coupled amplifier. Since the impedance of the retard coils increases with increase of frequency the higher frequencies will, in general, be

amplified more than the low. However, it is impossible to make retard coils without a certain amount of distributed capacity, the shunting effect of which tends to limit the amplification of the higher frequencies. By the proper design of coils it is possible to construct a retard-coupled amplifier which will give practically uniform amplification, e.g. throughout the speech range of 200 to 3,000 cycles. It is customary to make the retard and choke coils of the toroid or closed core type.

21. *Transformer Coupled Amplifiers.* From a theoretical point of view the transformer coupled amplifier (Fig. 26) should be the ideal type. By the proper choice of transformers it should be possible to match stages with respect to one another in such a way as to

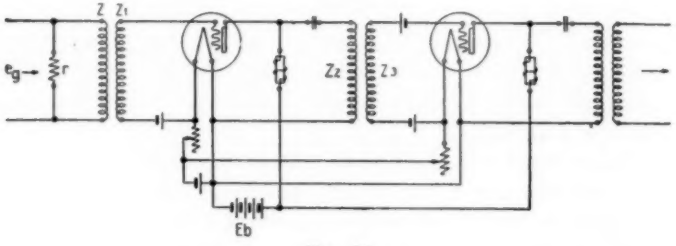

FIG. 26

obtain the greatest efficiency from tubes and batteries. The chief advantage of transformer coupling lies in the fact that the input voltage to the second stage may be made greater than the voltage output of the first, and so on at the same time that each tube operates into a properly matched impedance to give maximum power output. When uniform amplification over a relatively wide band of frequencies is not required, the interstage transformer may be designed to step up the voltage as many as 30 to 40 times. Other advantages are the economical use of plate batteries (Sec. 14) and the elimination of grid condenser and grid leak or high voltage grid battery.

However, the difficulties attendant upon the design and making of transformers are such that to realize the apparent advantages of this type of circuit will require very careful planning. This will be illustrated by the following example of a transformer to handle the frequencies of speech. Given an interstage transformer as in Fig. 26*a*, we will assume that the transformer works out of a tube impedance r_p and into a grid circuit impedance which has infinite resistance. Then,

imagining for the moment that the condenser C_1 has been removed, the output voltage e_2 of the transformer is

$$e_2 = \frac{emp}{r_p + jl_1p}, \tag{8}$$

in which e is the input voltage, l_1 is the inductance of the primary winding, m is the mutual between the windings, and p is 2π times the

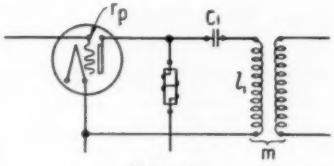

FIG. 26a

frequency. This neglects resistance of the winding and also capacity effects. Inspection of Equation 8 shows that e_2 varies with the frequency or with p in the manner shown in curve A, Fig. 26*b*, from which it is seen that the transformer tends to suppress the lower frequencies.

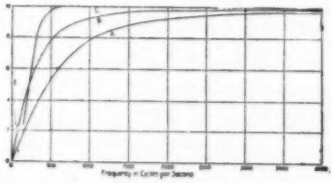

FIG. 26–b. *Curve A corresponds to l=1 henry: Curve C to l=2 henries, C_1= .141 f.*

Curve A shows the performance of a transformer as calculated from Equation 8 assuming $r_p = 5{,}000$ ohms and $l_1 = 1$ henry. Such a transformer would be quite unsuited for a speech frequency amplifier as it introduces very serious distortion below 1,000 cycles. Curve B is calculated on the assumption that $l_2 = 2$ henries and shows marked improvement over A for the lower frequencies.

In input transformer design it is ordinarily necessary to limit the inductance of the two windings not only because of the limited winding space but also because of the need of keeping down the capacity between windings and the capacity within each winding. Curve C shows the performance of the same transformer as Curve B when the capacity C_1 (Fig. 26*a*) is put in the primary circuit, C_1 having a

value of .141 m.f. and being so chosen as to tune l_1 to 300 cycles. With the capacity present Equation 8 becomes

$$e_2 = \frac{emp}{r_p + j\left(l_1 p - \frac{1}{C_1 p}\right)}. \tag{8A}$$

Use of the capacity improves the transformer characteristic for all frequencies above about 200 cycles and the combination therefore gives better results in a speech amplifier than the transformer alone.

The effect of distributed capacity in the windings (present especially in the secondary because of its greater number of turns) is, more or less, to shunt the high frequencies. This may be counteracted either by the inductance in the primary winding or, if this is not sufficient, by insertion of the proper inductance in series with the primary. It may be said, in a general way, that the lower the ratio of a transformer the better suited its frequency characteristic will be to a wide band of frequencies such as occurs in speech, and transformers with a ratio of 1 to 4 are made which require no correcting provided they are properly chosen with respect to the impedance characteristics of associated tubes.

The selective amplification of an amplifier for particular frequencies may be increased by tuning one or more of the secondaries of the interstage transformers with condensers. (See Equation 8A.)

Due to the fact that there is an appreciable distributed capacity between the primary and the secondary windings, an interstage transformer supplies capacity coupling as well as inductive coupling between adjacent stages, the phase of the capacity coupling being independent of the direction of winding while the inductive coupling is not. Therefore, the transformer may be so placed in the circuit that these two effects either aid or oppose one another. In order to secure the greatest amplification they should aid.

The transformer used in speech frequency work is, in general, made with an iron core; therefore, care should be taken to prevent the d.c. component of I_p magnetizing the core and reducing its efficiency. One method of accomplishing this is shown in Fig. 26 in which the d.c. component of I_p is bypassed by a choke coil. In circuits in which two or more interstage transformers are used, attention should be paid to the danger of magnetic feed-back. This can be largely eliminated by using transformers with closed magnetic circuits. Both the toroid

type core and the shell type core (commonly employed in power tranformers) have been found satisfactory, and especially the latter.

22. *Amplification of Higher Frequencies.* The transformer coupled amplifier is the type perhaps best suited to use at frequencies higher than those of speech. As a special case the amplifier circuit of Fig. 27 will be considered first. This circuit contains a tuned output and

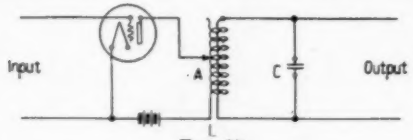

FIG. 27

should be used only in case a single frequency or very narrow band of frequencies is to be amplified but in this case will be found very satisfactory. The inductance L may consist of two parallel windings, insulated from each other to avoid any conductive connection between the plate battery and the output. Such an arrangement would be desirable if the output went to a detecting tube or another amplifying tube. By using a variable condenser C, the frequency of maximum amplification can be readily shifted but for any one setting the amplification will be as shown by Curve A of Fig. 28. For maximum output, the position of the tap A should be set so that the im-

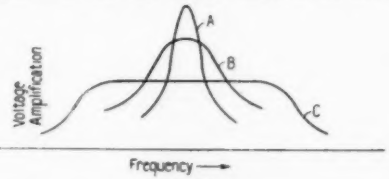

FIG. 28

pedance of the tuned circuit LC as seen from the tube is equal to the output impedance of the tube. Tuned circuit amplifiers are especially adapted for amplification at very high frequencies (above 2,000,000 cycles), where the effect of the capacities between the elements of the tubes makes other types of amplifiers very inefficient.

The amplification curve A will be broadened when more and more turns are added to the inductance L and the capacity of the tuning condenser C is diminished due to the effect of the distributed capacity of the coil. Curve B gives the amplification for a circuit where the condenser C is omitted in which case it becomes a retard coupled amplifier. Maximum amplification occurs at the natural frequency

of the coil including the capacity effects of the leads and elements of the tubes.

In case a relatively wide band of frequencies is to be amplified, transformer coupling is usually resorted to, and given suitably designed transformers one to two octaves can be amplified with very fair uniformity at frequencies between 100,000 to 2,000,000 and four to five octaves at frequencies below 100,000 cycles. Use of transformer coupling will broaden the characteristic of the amplifier as shown in curve *C*, Fig. 28, the exact shape of this curve being largely dependent upon the design of the transformers employed.

For frequencies up to about 100,000 cycles, transformers with iron cores of the ring type are suitable and are preferably enclosed in metal covers which are grounded. A transformer suitable for frequencies higher than 100,000 cycles may consist of two choke coils (one to two inches in diameter) of very fine wire, these coils being

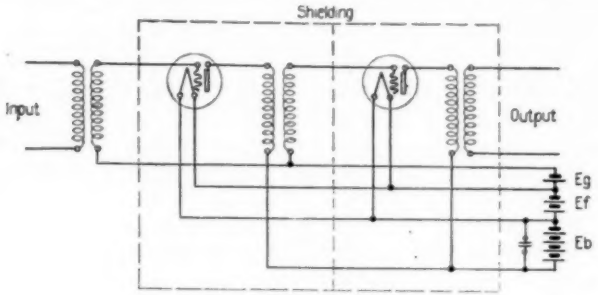

FIG. 29

mounted close together on a suitable form. The natural frequency of the coils will approximately determine the middle of the band of frequencies which are amplified and the coupling between the two coils will determine the width of the band, closer coupling resulting in a wider band. The coupling is generally a combination of electrostatic and electromagnetic coupling and therefore, in connecting all transformers for high frequency uses, it is essential to establish the proper phase relations between them (see paragraph 21). Each stage of the amplifier should be shielded as shown in Fig. 29 although in certain cases it may be dispensed with. The shielding should consist of brass or copper sheeting made into boxes with well-soldered joints and tightly fitting covers. Holes through the shielding should be just large enough to pass the insulation of the wires.

A common plate battery may safely be used for four or more stages provided a condenser is placed across the terminals of the battery as shown in Fig. 29. This condenser should have a capacity large enough to offer practically no impedance to the high frequency currents, and its use may be desirable although the plate battery is common to but two stages. Use of a common grid battery, as shown, introduces a small feed-back from the second stage to the first. This feed-back may be either positive or negative, depending upon the phase relations in the intermediate transformer and may be eliminated by placing a condenser across the grid battery terminals.

It has been the practice in high frequency amplification to use tubes with μ's between 6 and 10, interstage transformers being selected to step up the voltage as much as is possible consistent with the desired flatness of the amplifier characteristic. In general, the larger the ratio of the transformer, the more pronounced is the peak of the characteristic. Other things being equal, the most suitable tubes are those with the smallest internal electrostatic capacities. The largest of these capacities, in general, is that between grid and plate and tubes have been produced in which this does not exceed 5 $\mu\mu$ f. and in which the internal plate resistance is about 20,000 ohms.

In amplifying the higher frequencies the feed-back which occurs through the tube may require attention. In section 13, it was pointed out that an inductive output for a tube gives rise to a negative resistance characteristic in the input which means that feed-back is occurring. To eliminate the possibility of singing and also to eliminate unequal amplification of different frequencies which feed-back introduces, various means of neutralizing it have been proposed.[30] One such means is illustrated in Fig. 30 which is drawn to show radio reception with a loop antenna. Note that the grid of the first tube is joined to one end of the loop and the plate is joined to the other end through the balancing condenser C_b, the filament being joined to the midpoint of the loop. When C_b is chosen equal to C_3 the capacity between grid and plate, it is evident that the feed-back occurring through the tube is just balanced by that occurring through C_b. By adjusting the condenser C_b so as to permit of feed-back, very large amplification may be obtained at a single frequency but at the expense of flatness of characteristic.

[30] See Patent No. 1,183,875 issued to R. V. L. Hartley, and Patent No. 1,334,118 issued to C. W. Rice.

23. *Feed-Back Amplifiers.* This amplifier may be either resistance or inductive coupled, a typical resistance coupled circuit being shown in Fig. 31. In a feed-back circuit, attention must be paid to phase

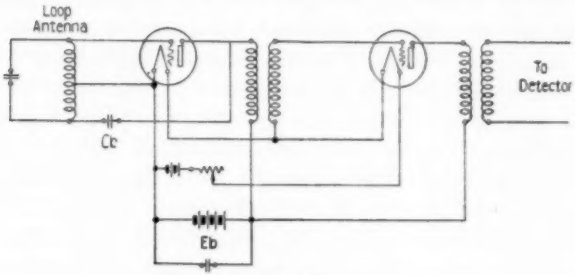

FIG. 30

relations. In Fig. 31, let the arrow along the resistance R_1 represent an increase in electron current to the grid of tube A. This corresponds to an increase in the potential of this grid. In phase with this increase in potential is an increase in electron current in R_2 as shown by the arrow. This, in turn, corresponds to a fall in potential of the grid of tube B and therefore to a reduction of the I_p in B, as indicated by the arrow at R_3, which produces an increase in I_p in C. Therefore, in this particular circuit the correct phase relations require the

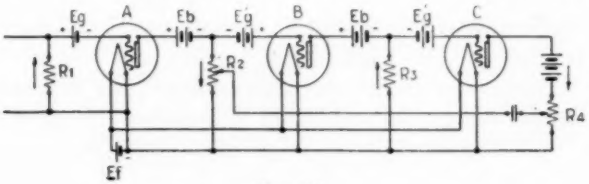

FIG. 31

output of one tube to be returned to the input of the second preceding tube or one of its alternate preceding tubes. The amount of energy fed back can readily be controlled by varying the portion of R_2 through which the feed-back current flows.

24. *Push-Pull Amplifier.* See Fig. 32. This type of circuit is particularly useful as a terminating stage since it makes possible the use of a low impedance in the output circuit without serious distortion. If the tubes A and B have identical characteristics, it is readily seen that the coils of the output transformer may be so connected that the fundamental and odd harmonics will aid one another, while all even

harmonics will oppose. Since the third and higher harmonics (counting the fundamental as first) are very small compared to the second, this circuit gives very nearly distortionless amplificat on. In speech amplifiers it perm'ts of considerable overloading without this being apparent n the quality of the output.

By reversing the transformer connections it is possible to cause the circuit to add the even harmonics and give the differences of the odd.

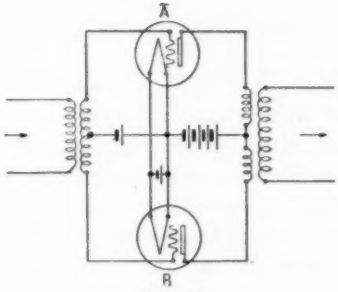

FIG. 32

An additional use for this circuit will be pointed out in the section dealing with modulation.

Fig. 33 shows a special type of push-pull circuit which is particularly adapted to the amplification of steady and low frequency voltages. It consists of a Wheatstone bridge in which two similar tubes form one pair of arms. The output circuit is the branch in which the galvanometer is ordinarily placed. When a voltage is applied

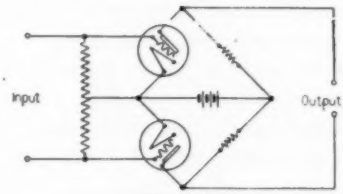

FIG. 33

to the two tubes in such manner that the potential of one grid is raised by the same amount as the grid of the other is lowered, the bridge becomes unbalanced and current flows through the output branch. For small applied voltages the amplification of the circuit is very nearly distortionless. The circuit has the obvious disadvantage

of requiring a close balance between the tubes and is therefore liable to require careful adjustment during use.

The push-pull circuit possesses one marked advantage over the resistance-coupled d.c. amplifier described in Sec. 18 for in it current flows through the output branch only when voltage is applied to the input. For the same reason the push-pull circuit is also useful for amplifying low frequency alternating voltages.

V. AMPLIFIER POWER SUPPLY

The proper power supply for amplifiers is an item of prime importance.

25. *Plate Voltage Supply.* The principal requirement placed on plate voltage is that it be steady. For this reason storage batteries are usually best, but good dry cells are more often used and when fresh, prove very satisfactory. The principal trouble encountered in the use of dry cells arises from an attempt to use partially rundown cells. A dry battery should be tested periodically for voltage, the reading being taken while the battery is delivering a current at least as large as that drawn by the amplifier. Whether dry cells or storage cells are used for plate voltage, in general not more than four stages should be operated from a single battery. In the case of a retard coupled amplifier whose stages are tuned with condensers, each stage should preferably have a separate plate battery to reduce the tendency to "sing."

A generator as a source of plate voltage is frequently used for power amplifiers. In case a direct current generator is used, a filter is generally necessary in the plate circuit to remove commutator ripples.

26. *Filament Voltage Supply.* A source of constant filament voltage is not necessary in order to insure constant space current within the tubes at temperature saturation, but in general any variation in filament current will affect the relative potential difference between filament and grid, and is, therefore, equivalent to a variation in the input voltage. This possible source of trouble must be particularly guarded against in such a circuit as that shown in Fig. 25, in which a portion of the adjustable resistance of the filament circuit is included between the filament and the "common point" A. If storage batteries are available, they form the best source of filament current; generators have been used satisfactorily however.

27. *Sources of Grid Potential.* A flow of electrons to the grid of a tube is liable to result either in distortion or a loss of amplification or

both (see Secs. 15 and 16). The steady negative grid potential required to prevent the input voltage carrying the grid to a positive potential may be obtained from either one of two sources: by a grid battery or by an IR drop in some resistance in the circuit. The requirements for a grid battery are very light since it is called upon to give no appreciable current. As was pointed out in Sec. 26, use of an IR drop for grid voltage pre-supposes steady filament or plate battery according to circumstances, and since the proper grid battery is readily obtainable the use of an IR drop is likely to prove desirable only in very unusual circumstances.

VI. TROUBLES IN AMPLIFIER CIRCUITS

28. *Noise.* The noise in amplifier circuits is due to several causes which may, in general, be grouped into two classes. Certain noises originate within the tubes and other noises find their origin in the circuit. The amount of noise in any amplifier limits the minimum input voltages which it will handle satisfactorily, for obviously input voltages which produce output currents of the same order of magnitude as the currents giving rise to noise will not be satisfactorily amplified.

29. *Tube Noises.* Tubes may be responsible for three distinct kinds of noises. (a) Ringing or rattling is due to the vibration of the tube elements and may be eliminated by proper tube construction or by some form of vibration proof suspension for the early stages of the amplifier. (b) Crackling may be produced by high resistance films on the inner surface of the bulb, forming conducting paths between the leads. Faulty electrical contact between the plate, grid and filament and their respective leads is also a frequent source of crackling. Furthermore, in tubes which are well constructed in regard to the points just mentioned, but which contain tungsten filament, crackling may be observed. This trouble is not found in all tungsten filament tubes but, when present, is sufficiently marked to become apparent in a two stage amplifier. (c) In carefully constructed amplifiers of more than three or four stages a noise which can best be described as a hissing or sighing is certain to be present. It appears to be related to an unavoidable statistical variation in the escape of electrons through the grid to the plate. Its magnitude has been found to correspond approximately to an output voltage from the first stage of between 5×10^{-7} volts and 5×10^{-6} volts. Between these limits the noise is found to increase as the output impedance of the first stage

is increased, and also to increase as the resistance across the terminals of the input increases. Its components, above 300 cycles, appear to be of about equal magnitude and uniformly distributed. It is, therefore, impossible at the present time to build amplifiers to handle voltages of less than this order of magnitude at any rate when the frequencies involved are in the audible range.

30. *Circuit Noises.* In general, circuit noises in amplifiers are due to one or more of the following causes: variations in grid and plate batteries, loose contacts and variations in resistances, leakage of condensers, and leakage across the insulating mounting upon which the amplifier parts are fastened, and external electric or magnetic fields acting inductively on the circuit. The remedy in each case is obvious once the exact cause has been found. To eliminate inductive effects in the wiring it is usually sufficient to run wires in pairs and to shield them electrically, the shielding being grounded. In arranging the various parts of an amplifier, it is well to place the bulky pieces at points in the circuit at which they will have as near zero potential as possible.

31. *Singing.* Singing, which is one of the most serious troubles in amplifiers, is always due to some form of feed-back. This may be magnetic, electrostatic, or in the form of mechanical vibrations as in an amplifier having a microphone attached to the input and a receiver to the output. Mechanical feed-back can also occur in the case of tubes whose parts can easily be set into vibration and a cure is usually found in some form of vibration-proof mounting. The coupling which is responsible for feed-back may be difficult to locate, but when found can usually be removed. Both retard coils and transformers may afford an easy method of coupling due to stray fields. If the coupling induces voltages which are in phase with the input voltages, it may cause singing, and if out of phase, the amplification may be seriously reduced. Closed core coils and magnetic shields are the usual remedies for this condition, although a rearrangement of the circuit parts may be necessary.

Certain kinds of electrostatic feed-back may be removed by enclosing each stage in a separate grounded metal cage or box. The electrostatic coupling due to tube capacities (Sec. 13) cannot be eliminated but it is possible to so design circuits that trouble from this source will not present itself. Thus an inductive impedance in the output circuit may prove troublesome because it induces a *negative* resistance back in the input circuit; a non-inductive output can never

do this. Feed-back through tubes increases with frequency, and in the case of high frequencies, it may sometimes be necessary to use resistance coupled rather than reactance coupled circuits.

32. *Blocking*. Two entirely different types of blocking may occur in an amplifier. They both result from the grid of one or more tubes having been carried to a positive potential by the input voltage. While positive, the grid picks up a negative charge of electrons which is removed more or less rapidly by the grid leak. In case the leak resistance is high, a residual charge may remain upon the grid for an appreciable length of time, depressing its mean potential to so low a value that the output of the tube is cut to zero or very nearly zero. The remedy is obviously to reduce the input voltage or to increase the the voltage of the negative grid battery. In certain cases, a readjustment of the resistance of the grid leak may be desirable.

The second type of blocking involves secondary emission from the grid as discussed in Sec. 9. It can occur only when the input is sufficient to force the grid potential of some tube positive by as much as 10 or 15 volts, and then if the grid leak resistance is large enough, secondary emission will hold the grid at about this positive potential and entirely prevent proper functioning of the amplifier. In eliminating this type of blocking, the first step should be to note the effect of increasing the filament currents as secondary emission is less likely to occur when the filament yields a copious supply of electrons. If this does not remove the trouble, the negative grid batteries in the stages at fault may be increased and lower grid leaks may be desirable. The volume of input to each stage should also be considered.

33. *Distortion*. Distortion in an amplifier circuit may result either from a failure to amplify all frequencies by the same amount or from the generation of overtones of the fundamental frequencies in the input.

The unequal amplification of various frequencies arises from the presence of resonant characteristics in the circuit. This may take the form of a feed-back which discriminates in favor of certain frequencies, the feed-back not being pronounced enough to cause singing. A negative feed-back may also occur, causing a loss of efficiency over some particular frequency range.

The distortion which arises from the generation of overtones is due to non-linear voltage-current characteristics in one or more branches of the circuit. The usual sources of this trouble are curvature of the plate and grid characteristics (See Equation 4) and the variable

permeability of the iron used as cores. With properly chosen coils, practically distortionless amplification can be secured by the method indicated in Sec. 12. In general, to accomplish this, the output impedance need not be more than two or three times r_p. In case it is necessary to use a low output impedance in the final stage, distortion may be reduced by using the push-pull circuit of Sec. 23.

In using an amplifier under circumstances such that distortionless output is desired, care should be taken that no tube by itself is overloaded or caused to work in such fashion that its dynamic characteristic is curved. Distortion which arises from curvature of this characteristic can be detected by inserting an ammeter in the plate circuit of each tube. When each characteristic is straight, or nearly so, there should be no change in ammeter reading as the source of input voltage is thrown on and off, and in the case of a variable input such as that arising from speech, the ammeter readings should remain constant while the amplifier is in operation. This test will not detect distortion which arises from selective amplification with respect to frequency.

34. *Calculation and Measurement of Amplification.* Provided all parts of an amplifier circuit are functioning properly and its constants are known, its amplification can be calculated quite accurately. The following example will illustrate the procedure to be followed in any case. Referring to the transformer coupled amplifier of Fig. 26, assume that the ratio of the first input transformer is $z:z_1$ and that the ratio of the second input transformer is $z_2:z_3$; assume also that z_2 is numerically equal to r_p, the plate circuit resistance of the first tube, r_p being also the plate circuit resistance of the first tube. Then, calling e_g the input voltage, the voltage across the first tube is

$$e_g\sqrt{\frac{z_1}{z}};$$

the voltage across the primary of the second input transformer is

$$e_g\frac{\mu}{2}\sqrt{\frac{z_1}{z}}$$

since z_2 is numerically equal to r_p; and across the secondary is

$$e_g\frac{\mu}{2}\sqrt{\frac{z_1}{z}}\sqrt{\frac{z_3}{z_2}}.$$

Hence the voltage amplification of this portion of the amplifier is

$$\frac{\mu}{2}\sqrt{\frac{z_1}{z}}\sqrt{\frac{z_2}{z_2}},$$

and a similar argument applies to the following stages.

The measurement of amplification can be accomplished by the obvious procedure of determining the magnitudes or the relative magnitudes of the input and output current. This can be done for either a single stage or for several stages at once.

A very satisfactory circuit for measuring amplification is illustrated in Fig. 34 and through use of a vacuum tube voltmeter (see Sec. 56) as a comparison means, it is capable of an accuracy of about 2%. The conditions which the resistances R_1 and R_2, etc., should satisfy are very simple. The network connected between the oscilla-

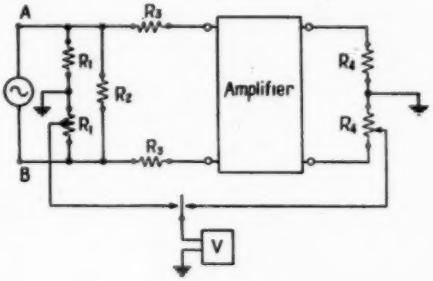

FIG. 34

tor and amplifier input should present an impedance, looking into it from the right, equal to the input impedance of the amplifier; likewise the output impedance of the amplifier should equal $2R_4$. The input impedance of the vacuum tube voltmeter is so high as not to shunt the resistances across which it is connected appreciably. The grounds at the mid points eliminate the disturbing effects of capacities to ground. Under these conditions the voltage amplification a is given by the equation:

$$\frac{a}{\beta}\left[\frac{4R_3(2R_1+R_2)+2R_1R_2}{2R_3(2R_1+R_2)+2R_1R_2}\right],$$

in which α, β are the fractions of R_1 and R_4 respectively, across which the voltmeter is connected to obtain equal readings when the switch W is thrown from one position to the other. In case R_2 is made quite small with respect to R_1, R_3, the expression for a reduces approximately

to $a=\frac{2a}{\beta}$. An expression for current amplification can readily be derived.

Another simple measuring circuit is shown in Fig. 34a in which O is an oscillator of the desired frequency, F is a filter ro remove harmonics from the oscillator current, R_1R_2 and R_3R_4 are attenuating networks consisting of resistances, WWW are switches by which the telephone receiver T can be joined either to the output of the amplifier

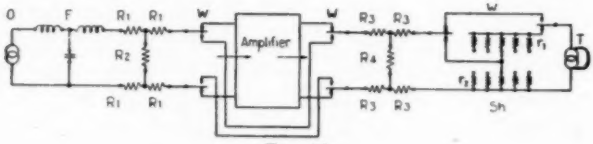

FIG. 34a

or directly to the oscillator, and also provide means for removing the known attenuation in the shunt *Sh* (receiver shunt) at the same time the amplifier is removed. By the proper design of the receiver shunt, which will be discussed presently, the attenuation required to give the same volume of sound in the receiver whether the amplifier is in or out may be read directly.

In setting up the circuit of Fig. 34a, special attention must be given to the networks R_1R_2 and R_3R_4. In addition to reducing the input to the amplifier to a value safely below the overload point, R_1R_2 should be designed to present an impedance (when seen from the amplifier) equal to that out of which the amplifier is to operate in service. Otherwise the measurements of amplification may be without significance.

The network R_3R_4 serves two important purposes. It is designed to present toward the amplifier the same impedance as the amplifier is to work into in service, and this in turn requires that the input and output impedances of the amplifier be practically equal (or if not, then small with respect to R_1) for otherwise the network R_3R_4 when joined to R_1R_2 will not draw the same fraction of current as the amplifier, thereby upsetting the comparison upon which the measurements are based. Furthermore, the attenuation in R_3R_4 is to be sufficiently large that variations in the impedance of the receiver and its shunt as seen from R_3R_4 will not appreciably affect the impedance into which the amplifier works as the receiver shunt setting is changed. A simple calculation will show how great the attenuation must be in any given case to satisfy these conditions.

Proper values for the steps of the receiver shunt may be calculated as follows, reference being made to Fig. 35 in which the currents and potentials indicated are in accordance with the assumptions made

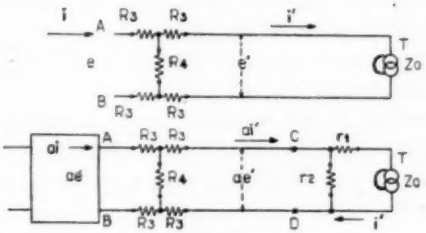

FIG. 35

regarding the attenuation in the various portions of the circuit. Calling a the amplification to be measured it may readily be shown that

$$\frac{(a-1)^2}{a}=\frac{r_1}{r_2}. \tag{9}$$

Or if R is the impedance of the network R_3R_4 as seen from the receiver, and the shunt Sh is proportioned so that it also presents the impedance R to the receiver, whence

$$R=r_1+\frac{r_2R}{r_2+R},$$

then Equation (9) gives

$$a=\frac{R+r_2}{r_2}.$$

Taking account of the necessary approximations it is readily possible to measure current amplification to within 5%, for a range of frequencies extending from 200 to 3,000 cycles. Receiver shunts are made which, in 10 to 15 steps, will reach a maximum reduction ratio in current of 25:1 which corresponds to an energy reduction of 625:1, and this does not represent the greatest range possible.

In case a rougher approximation of the amplifying power is sufficient, the circuit of Fig. 34a may be simplified by omission of the network R_3R_4 and reversal of the receiver shunt to present a constant impedance (except for variation of impedance with frequency and phase angle) toward the amplifier. The network R_1R_2 should preferably be retained and should be so proportioned that the current through the right hand R_1 branches is practically the same whether connected with the amplifier or directly to the receiver.

In measuring the over-all amplification of a multistage circuit, it will probably be desirable to add fixed but known attenuation units similar to R_3R_4 to the receiver shunt which may be cut in or out as required. These units may be given an attenuation equal to and twice the total attenuation of the shunt, etc., after the fashion of the ordinary resistance box. In constructing attenuation networks the arrangement indicated in Fig. 34a will be found desirable in that the symmetrical placing of the branches tends materially to eliminate errors which might otherwise arise due to capacities to ground in the oscillator and amplifier. Pairing of lead wires and shielding of leads and resistance coils will be found desirable for accurate work.

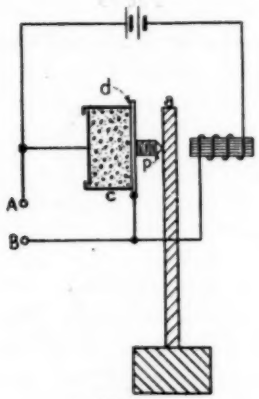

FIG. 36

The filter F should be used in case the amplifier tends, because of the limited range of frequencies which it passes or because of some other kind of distortion, to modify the quality of the note given by the oscillator, it being very difficult to match sounds for intensity which differ in quality.

A very satisfactory type of audio-frequency generator is shown in Fig. 36; it is a buzzer which operates, not by making and breaking current, but by varying it periodically with a microphonic button. The vibrating parts of this generator may be tuned to any audio-frequency, e.g. 800 cycles, and it gives quite accurately a sinusoidal variation of current, although it is customary to insert a filter (Sec. 6) to insure the input energy being accurately of one frequency.

VII. THERMIONIC MODULATORS

In discussing modulation the terminology which has been developed in connection with radio and carrier-current signaling will be used.

By the term "modulation" is meant the varying of the amplitude of a relatively high frequency wave, so that its envelope represents a particular low frequency wave or combination of such waves. (See Curves A, B and C, Fig. 37). The combination of low frequency modulating waves may be very complicated, as in the case of speech, but the principle involved is common to all cases of modulation and can be clearly brought out by the consideration of a single low frequency.

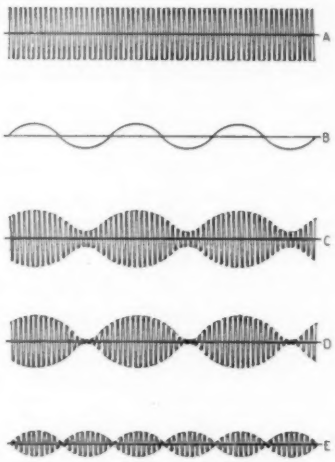

FIG. 37

Let Fig. 37, C, represent a high frequency wave modulated by a sinusoidal low frequency. The wave C can be represented by

$$y = a(b + \cos qt) \cos pt, \tag{10}$$

in which $\frac{p}{2\pi}$ is the high (carrier) frequency and $\frac{q}{2\pi}$ the low (signal) frequency. Equation 8 can be rewritten in the form

$$y = ab \cos pt + \frac{a}{2}[\cos (p-q)t + \cos (p+q)t], \tag{11}$$

which brings out the fact that the modulated wave C contains, in

general, three distinct frequencies—the carrier frequency $\frac{p}{2\pi}$, a difference frequency $\frac{p-q}{2\pi}$, and a summation frequency $\frac{p+q}{2\pi}$. These latter frequencies represent the so-called "side bands" of the modulated wave.

Two special cases of the wave represented by Equations 10 and 11 are represented graphically at D and E of Fig. 37 and correspond to $b=1$ and $b=0$ respectively. When $b=1$ it is evident that the amplitude of each side band is half the amplitude of the carrier frequency; such a wave is said to be "completely modulated"; when $b=0$ the carrier frequency $\frac{p}{2\pi}$ is absent altogether.[31]

35. *Means for Producing Modulation.* Perhaps the simplest case of modulation is that illustrated by continuous-wave radio telegraphy, in which the intermittent radiation of a uniform wave is accomplished by means of a telegraph key. In most cases, however, modulation requires a gradual change in the amplitude of the high frequency wave. For effecting this the vacuum tube possesses two properties which make it particularly useful—(a) the E_g,I_p characteristic is very nearly parabolic (Sec. 8); (b) the current in the plate circuit is a function of the grid potential (Fig. 10).

Circuits, by means of which modulation may be effected by each of these properties, are described in the following paragraphs.

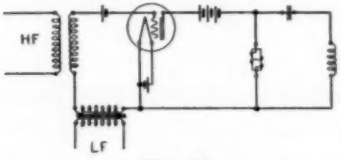

FIG. 38

36. *Modulation by Curved Characteristic.* Considering the circuit of the type illustrated in Fig. 38, let it be assumed that a voltage

$$e=A\cos pt+B\cos qt$$

is applied to the input of the tube. The result is shown graphically in Fig. 39. When this value of e is substituted in Equation 4 we

[31] For a more complete discussion of modulation and the nature of the side bands, see R. V. L. Hartley, Procd. Inst. of Radio Engrs., Feb., 1923, or Bell System Technical Journal, Apr., 1923.

obtain for the modulated output (i.e., terms whose frequencies are of the order $\frac{p}{2\pi}$).

$$J_m = A\left[\frac{\mu}{r+r_p} + \frac{\mu^2 r_p r'_p}{(r+r_p)^3} B\cos qt\right]\cos pt. \qquad (12)$$

As pointed out above, the first term gives the carrier wave and the second term, the two side bands. It will be remembered that this

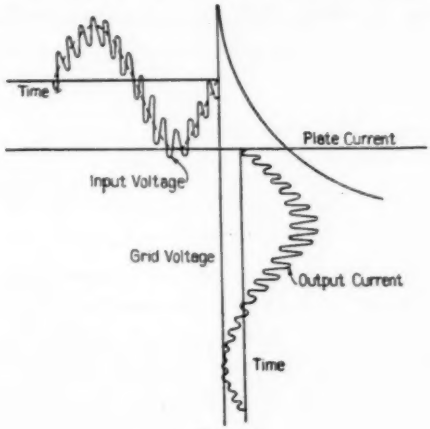

FIG. 39

equation neglects terms of higher order than the second, which is permissible, so long as the tube characteristic is approximately parabolic.

Certain points regarding Equation 12 should be noted. In the first place, the amplitude of the side bands is proportional to the product AB and is therefore, independent of the relative amplitudes of the original carrier and modulating frequencies. Also the modulated current is proportional to the first power of B, the amplitude of the low frequency wave; i.e., although the modulation is effected by the curvature of the tube characteristic, the modulated output is free from distortion. Furthermore, the modulated output voltage is proportional to $\frac{r}{(r+r_p)^3}$ which is a maximum when $r=\frac{1}{2}r_p$; and the modulated output energy is proportional to $\frac{r}{(r+r_p)^6}$ which is a maximum when $r=\frac{1}{5}r_p$.

Another type of circuit in which the modulation is dependent upon the curvature of the E_g,I_p characteristic is shown in Fig. 40. The two tubes are supposed to be alike; so long as no low frequency is impressed on the grids the high frequency space currents are equal, each passing through one of the primary coils of the transformer T_0, the order of winding these coils being such that this condition gives zero current in the secondary. However, the presence of a low frequency voltage (*L.F.*, Fig. 40) raises one grid potential at the same time that it losers the other, with the result that the high frequency

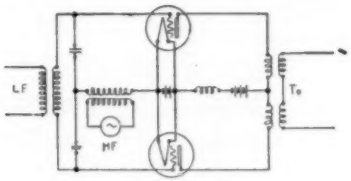

FIG. 40

currents in the two primary coils are no longer equal, and a high frequency current therefore flows in the secondary of T_0, the amplitude of which is determined by the degree to which the two tubes are unbalanced by the low frequency input. It is apparent that the output of this modulator circuit contains only the two side bands and none (or very little, if the tubes are not exactly identical) of the carrier frequency and therefore corresponds to curve *E* in Fig. 37. It is particularly useful in communication circuits where several telephone or telegraph channels are desired on the same pair of wires. Since only the side bands are transmitted the total current which must be handled by repeaters and other line apparatus is materially reduced. By the use of the proper wave-filter it is also possible to suppress one side band, thereby approximately cutting to one half, the width of the frequency band to be transmitted. As will be pointed out under homodyne detection, the suppressed carrier frequency must be supplied locally before detection can occur.

37. *Modulation Effected by Controlling Plate Current with Grid Potential.* Numerous circuits have been developed for modulation, making use of the fact that the grid potential affects the resistance of the plate circuit. Two circuits of this type are shown in Figs. 41 and 42. In the first, the plate circuit of the tube forms a shunt across a portion of the antenna inductance. As the grid potential is varied, the antenna is, therefore, thrown more or less out of tune,

with the consequent radiation of a variable amount of high frequency energy.

Fig. 42 shows one of the most efficient modulating schemes thus far developed. As the grid potential of the modulator tube *A* varies, causing a change in plate current through this tube, the plate voltage applied to the oscillator tube, *B*, fluctuates, because of the presence of the low frequency choke coil. Under this condition of variable plate-voltage the oscillator gives a variable amount of high frequency energy to the antenna. When used for speech modulation this circuit is very efficient and gives good quality. The tubes *A* and *B* are ordinarily of the same type.

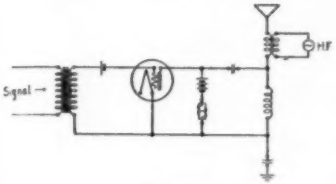

FIG. 41

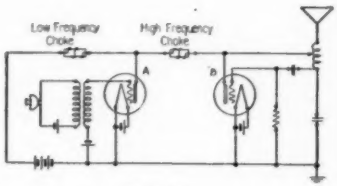

FIG. 42

Many other modulating circuits have been designed, and those given above are to be considered merely as illustrative of the general manner in which the properties of the vacuum tube may be applied to the problem of modulation.[32]

VIII. THERMIONIC DETECTORS

Line the modulator the detector is a device for the production and separation of difference frequencies. The object of modulation is, in general, to transform a high frequency $\frac{p}{2\pi}$ and a low frequency $\frac{q}{2\pi}$ into two high frequency side bands, $\frac{p \pm q}{2\pi}$. Detection accomplishes the inverse operation of forming from a carrier frequency $\frac{p}{2\pi}$ and either or both side bands the original low frequency $\frac{q}{2\pi}$, detection often being referred to as demodulation. Detection, like modulation, can be most readily described by the consideration of a single pair of frequencies.

[32] For other types of modulator circuits see a paper by R. A. Heising, Procd. Inst. Radio Engrs., Aug., 1921.

When carried out by means of a vacuum tube it results from rectification in either the grid circuit or the plate circuit. This rectification may arise either from unilateral conductivity or a curved current-voltage characteristic as pointed out in the following paragraphs.

38. *Detection by Curved Plate Characteristic.* Considering the circuit shown in Fig. 43 and assuming an input voltage $e = A(B+\cos qt)\cos pt$, it follows from Equation 4 that the output current, considering only those terms whose frequencies are of the order $\frac{q}{2\pi}$, is

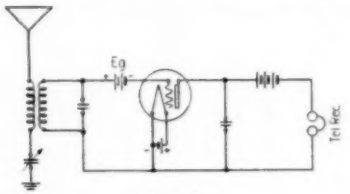

FIG. 43

$$J_d = \frac{1}{2!}\frac{\mu^2 r_p r_p'}{(r+r_p)^3} A^2(B\cos qt + \frac{1}{4}\cos 2\,qt). \qquad (13)$$

The current J_d, known as the "detected current," therefore, consists of a term whose frequency is $\frac{q}{2\pi}$ and another term whose frequency is twice this. The presence of these two frequencies is readily understood. The detected current of frequency $\frac{q}{2\pi}$ corresponds to the difference frequency (See Equations 10 and 11) of the carrier of amplitude AB and each side band of amplitude $\frac{A}{2}$ and is therefore proportional to $2\,\frac{A}{2}\cdot AB$. The second term of the detected current represents the difference frequency $\frac{q}{\pi}$ between the two side bands themselves and, as is to be expected, its amplitude is proportional to $\frac{A^2}{4}$. In case one of the side bands is suppressed before detection, this term of double frequency is entirely absent in the detected current. Furthermore, the amplitude of the detected current of frequency $\frac{q}{2\pi}$ is

independent of the *relative* amplitudes of the carrier wave and the side bands. In general, AB is large compared to $\frac{A}{2}$ with the result that the term of double frequency in the detecting current is negligible. It follows, therefore, as in the case of modulation that the detecting current is practically free from distortion.

The detecting action resulting from the curved plate characteristic is shown in Fig. 44.

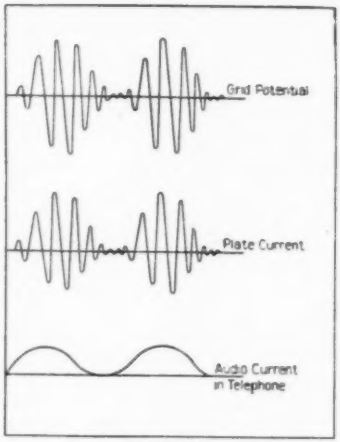

FIG. 44

Equation 13 leads to the result that the output voltage of a detector tube, when working as above, is a maximum when $r = \frac{1}{3}r_p$. In using these relations note that r represents the value of the output resistance for the high frequency and not the low frequency. When using an

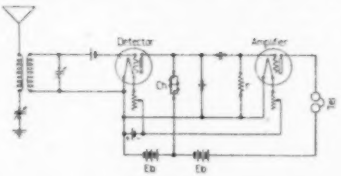

FIG. 45

amplifier on the output of a detector (see Fig. 45), it is important to choose r to give the maximum detecting *voltage*. As in amplifier circuits it is essential in this type of detector that the grid remain always negative. See also the sections on amplifiers.

39. *Detection by Rectification in Grid Circuit.* This type of circuit (see Fig. 46) is now in very general use for radio purposes, and is characterized by the grid blocking condenser C_s. Contrary to the

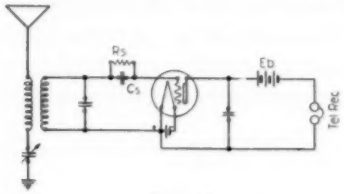

FIG. 46

preceding type of detector, the present requires the flow of electrons to the grid and works best when the grid is held permanently at a small positive potential. The action of the high frequency input causes the periodic accumulation of a negative charge upon the grid and the blocking condenser, thus lowering E_g and diminishing I_p. This action is clearly illustrated in Fig. 47. This circuit is most effective

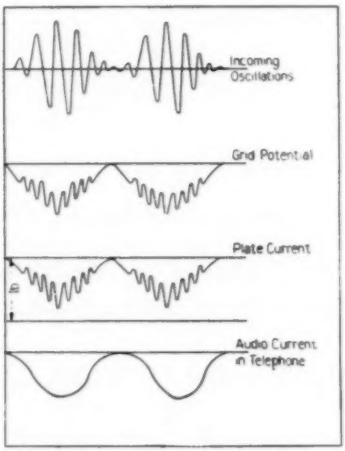

FIG. 47

when the carrier frequency is much greater than the signal frequency, and not as efficient as the circuit described in Sec. 38 when the carrier is say only four or five times as great as the signal frequency.

Attempts to deduce a quantitative relation for the detecting current in this type of detector have as yet met with little success, one of the principal reasons being that very little is known about the "dynamic"

grid current characteristic.[33] Experiments show, however, that the detecting current is practically proportional to the square of the input voltage, provided that this is small, thus establishing the relation,

$$J_d = ae^2,$$

which corresponds in form to Equation 13 above.

In designing this type of detector circuit, attention must be paid to the value of the blocking condenser C_s and its leak R_s. It is clear that the capacity of C_s should be sufficiently small to cause the grid to undergo the maximum potential change as a result of the relatively small electron charge picked up, and yet it must be several times larger than the tube capacity between grid and filament. Furthermore, the time constant of C_s and R_s should approximately match the frequency of the detected current. With the more common detector tubes and radio frequencies, capacities of the order of 200 $\mu\mu$f. are satisfactory.

Detector circuits with grid blocking condensers may be coupled to amplifiers as readily as the other type of detector, and, in general, a higher output resistance for the detector can be used, thus making possible more efficient coupling between the detector and the first stage of amplification. In increasing the output resistance of the detector it should, however, be borne in mind (see Sec. 13) that a secondary result is to reduce the input impedance of the detector, which may entail a reduced input voltage.

40. *Heterodyne and Homodyne Detection.* In continuous-wave radio telegraphy, the dots and dashes of the code are transmitted by a continuous carrier wave of a single frequency. *Heterodyne* reception consists in supplying a slightly different frequency at the receiving station, the transmitted and locally generated frequencies when applied to the detector acting exactly as the carrier and side band frequency described above. The useful output of the detector is the difference frequency which, of course, is chosen in the audible range.

It follows from Sec. 38 that the heterodyne detected current is proportional to the product of the amplitudes of the transmitted and locally generated waves. Because of this fact a feedback circuit may be used to advantage as a means of increasing the strength of both high frequency terms. In the usual type of feed-back detector, the detector tube is also used as the source of local high frequency. Such

[33] For a discussion of this topic see Hulbert & Breit, Phys. Rev., Nov., 1920, pp. 408-419; Oct., 1920, pp. 274–281.

a circuit is shown in Fig. 48. The oscillatory circuit O is tuned to differ in frequency from the incoming signal by an amount which will give a satisfactory difference frequency in the telephone receiver T. By varying the coupling at M, the intensity of the beat note can be readily changed. It must, however, be sufficient to cause the circuit as a whole to oscillate at the natural frequency of O. A feed-back arrangement is particularly applicable to those cases in which the carrier frequency is much higher than the signal frequency and is therefore generally used with a blocking condenser.

In telephone systems, whether radio or carrier current, it is frequently desirable to suppress the carrier frequency and transmit only

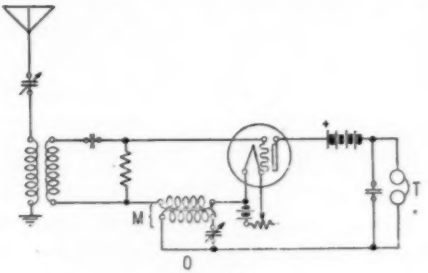

FIG. 48

one[34] or both of the side bands. Detection with only the side bands present would result in a double frequency detecting current which obviously would not be permissible in a telephone circuit. Whenever the side bands alone are transmitted, a locally generated high frequency exactly equal in frequency and phase to the original carrier frequency must be supplied.[35] This is known as *homodyne* reception.

41. *Measurement of Detection Coefficient.* The constant a in the relation $J_d = ae^2$ is called the "detection coefficient." Its measurement by direct means is not difficult but as seen from Equation 13 it involves so many factors that no satisfactory indirect methods of determination have been developed. The requirements of the direct method are quite obvious, and for circuit details reference is made to the *Thermionic Vacuum Tube* by Van der Bijl.

42. *Detecting Efficiency.* A knowledge of the detecting coefficient a tells very little about the detecting efficiency of a tube, the efficiency

[34] For a discussion of the advantages of *single* side band transmission see reference given in footnote 12.

[35] See J. R. Carson, Procd. Institute of Radio Engineers, *11*, p. 271; 1923.

being defined as the ratio between the low frequency energy in the output and the high frequency energy in the input. The efficiency involves the input impedance of the tube which is a function of the circuit constants as well as the tube. It is therefore impossible to specify the detecting efficiency of a tube without certain data concerning the circuit in which it is to be used; a is therefore without much significance.

43. *Detecting Coefficient and Plate Voltage.* The variation of the detecting coefficient with plate voltage depends upon the type of circuit. If detection is accomplished by a curved plate characterisic, experiment shows (see Fig. 49) that the operation is best when the

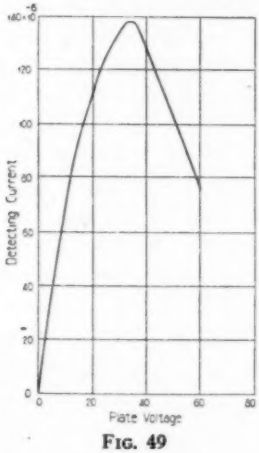

FIG. 49

effective voltage is about equal to the potential drop in the filament, it being presupposed (see Sec. 38) that E_g is enough less than zero to keep the grid negative at all times. If detection is accomplished by means of a blocking condenser, the variation is as shown in Fig. 50, no sharply defined maximum being present in the curve.

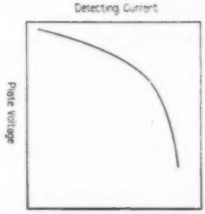

FIG. 50

The variations of detected current under heterodyne operation are shown in Figs. 51 and 52 which refer respectively to detection with and without a blocking condenser. The abscissa, e_1e_2, gives the product of the two high frequency amplitudes in the input. As is to be expected when no grid condenser is used (see Equation 13), the variation of detected current with e_1e_2 is very nearly linear provided E_g is sufficiently negative. Fig. 51 referring to detection with grid condenser shows no linear relation however. The data for Figs. 51 and 52 are taken from Van der Bijl.

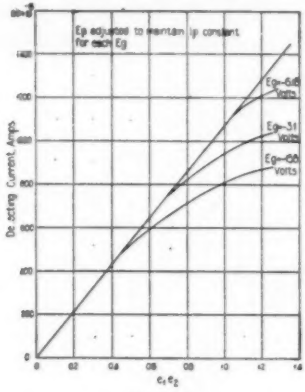

FIG. 51

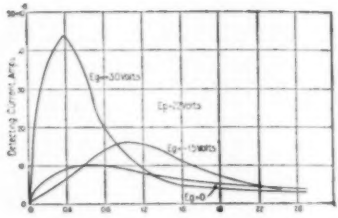

FIG. 52

44. *Comparison of Tubes as Detectors.* If a tube is available whose detecting coefficient is known, other tubes may be calibrated in terms of this standard. The comparison of detectors can be very readily carried out by means of such a circuit as shown in Fig. 53. This circuit makes use of a grid blocking condenser but could readily be

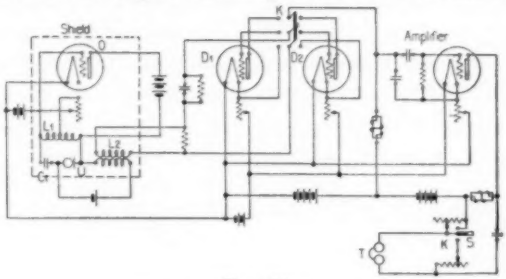

FIG. 53

rearranged not to employ it. In use, switches S and K are operated together in such manner that the receiver shunt is cut out when the

receiver is connected to the tube of lower detecting power. By proper adjustment of the shunt the two tubes can then be brought to apparent equality, the difference being read from the calibration of the shunt.

IX. VACUUM TUBE OSCILLATORS

As pointed out in the section on amplifiers, it is easy to design feed-back circuits which will sing, i.e., will generate continuous oscillations. The necessary requirements which an oscillating circuit must meet are two in number and are readily understood. Any small alternating voltage when applied to the input generates a current in the output, and by virtue of the feed-back a portion of this energy is returned to the input. For continuous oscillations, the energy returned must be in phase with the original input supply. Furthermore, letting e represent the initial input voltage, the feed-back coupling must be sufficient to return to the input a voltage greater than e. If it is less than e the circuit will amplify but will not oscillate.

The circuit requirements necessary for any given tube to return a voltage greater than e may readily be stated in mathematical form, but so far as the practical design of oscillators is concerned, this statement has no particular value. The design of circuits is still very largely an empirical matter, and the problem is not so much to make the circuit oscillate as to make it oscillate with the proper frequency, efficiency and output power. These requirements can usually best be met by trial and adjustment taking into account such general theoretical considerations as follow.

Vacuum tube oscillators make a convenient way of obtaining large high frequency currents at small voltages and large a. c. voltages at small currents for testing purposes.[36] A special oscillator giving a very pure sine wave is discussed in Sec. 53.

45. *Equivalent Resistance of the Oscillator Circuit.* Oscillator circuits are of many types, but the fundamental action of all of them can be reduced to common terms, and to simplify the discussion the type of circuit illustrated in Fig. 54 will be discussed.

It will be noted that a d.c. voltage is applied between filament and plate by means of a battery in series with a choke coil. This choke may be considered as having zero d.c. resistance and virtually an infinite resistance to the a.c. generated by the oscillator. An oscillating circuit consisting of inductance, resistance and capacity is also connected between filament and plate, and by means of another in-

[36] See W. C. White, Gen. Elec. Rev., *20*, p. 635; 1917.

ductance joining filament and grid a portion of the output energy is fed back to the input.

In circuits as usually constructed the ohmic resistances r_1 and r_2 of the oscillating circuit are very small compared to the impedances of L_2 and C. It follows that the frequency of oscillation differs but slightly from the natural frequency $1/2\pi\sqrt{L_2C}$, and the oscillating circuit may therefore, be looked upon as introducing nothing more than a pure resistance (so far as the fundamental component of I_p is concerned) into the output circuit of the tube. Except for considerations of feed-back, we may therefore imagine the oscillating circuit

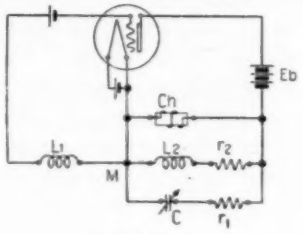

FIG. 54

L_2, C, replaced by an equivalent resistance whose value is given by the equation $R=\dfrac{r_1r_2+L_2/C}{r_1+r_2}$. This "equivalent resistance" is important in dealing with oscillating circuits.

46. *Phase Relations.* Typical phase relations between the various currents and voltages in the oscillating circuit are illustrated by the oscillograms in Fig. 55. Note that the oscillation current I_0, and also E_p and E_g show practically sinusoidal variations. Such variations will be found over very wide ranges of adjustment. Although I_p does not show a sinusoidal variation, the fundamental component (whose frequency is the same as that of I_0 is relatively much larger than the higher components.

Note that the variations of E_p and E_g are practically 180° out of phase, and also that E_p and the fundamental of I_p are 180° out of phase. This latter condition is obviously required for maximum power output by an a.c. generator. In order that I_p be a maximum when E_p is a minimum, E_g must be a maximum at this time.

47. *Dynamic Characteristic.* A tube when operating into an output resistance follows a dynamic characteristic (also called derived characteristic)[37] whose slope is somewhat less than that of the static (see

[37] See L. A. Hazeltine, Procd. Inst. of Radio Engrs., April, 1918.

Sec. 12). The dynamic characteristic is flatter than the static characteristic since the oscillating circuit L_2, C acts as an equivalent resistance. In fact, the dynamic characteristic of the oscillating tube may differ in one important respect from that of Sec. 12, for in an oscillator the grid potential has a relatively very high positive value

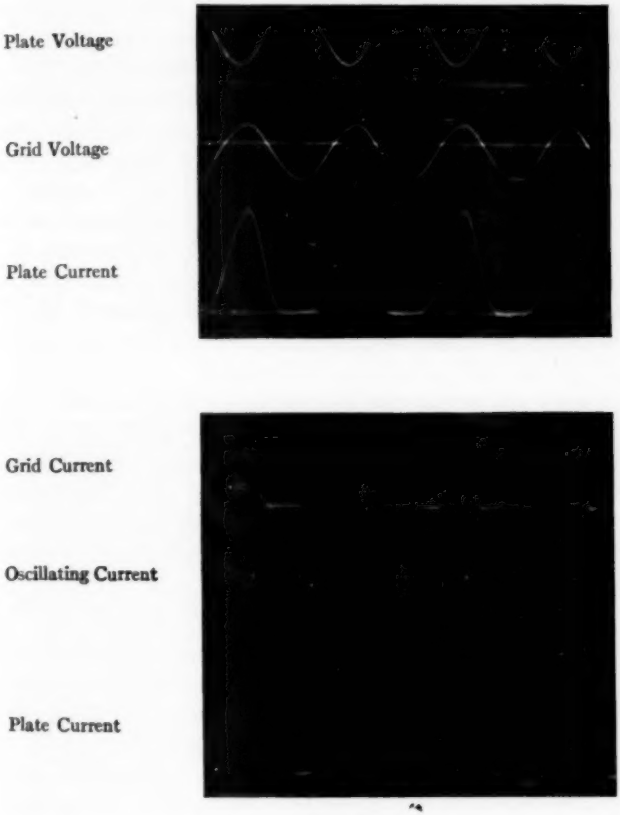

FIG. 55

for a portion of the cycle. Consequently the dynamic plate characteristic very frequently turns downward at its upper end (point B, Fig. 56). As will be pointed out later, this feature of the dynamic characteristic is likely to be one of the factors tending to limit the amplitude of oscillation.

48. *Amplitude of Oscillation.* As yet no very comprehensive formula has been derived, either theoretically or empirically, to express the amplitude of oscillation in terms of the tube and circuit constants and the applied E_g. However, certain general statements can be made which will serve as useful guides.

We shall not consider further the circuit shown in Fig. 54; it shows a negative grid battery in the grid filament branch, which, if sufficiently large to control the oscillator when in operation, usually makes the starting difficult and uncertain, and is therefore undesirable. To eliminate the grid battery and yet supply sufficient negative potential, once oscillations have started, circuits are usually supplied with a grid blocking condenser and high resistance leak (see Figs. 64 and 65). Since for a portion of each oscillation the grid is positive, it picks up a charge of electrons which in flowing off through the leak creates an average negative grid potential. It is apparent that as the amplitude of oscillation increases, the charge picked up at each positive swing of the grid potential increases, with the result that this average negative potential tends to sink lower. The importance of this control feature will be brought out presently.

It is generally found that the oscillations build up to such a value that the greatest positive potential of the grid, which will be represented by $E_{g\,max}$, becomes practically equal to the lowest plate potential, $E_{p\,min}$. It is not difficult to understand why this condition should represent a sort of limit. It is usually found that the dynamic characteristic, as shown at *B*, Fig. 56, tends to bend rapidly downward

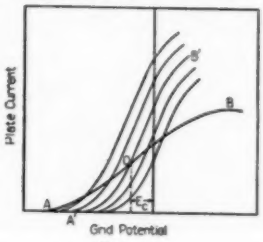

FIG. 56

as $E_{g\,max}$ becomes greater than $E_{p\,min}$. When $E_{g\,max}$ tends to become greater than $E_{p\,min}$ the electron current to the grid rises suddenly (see cathode ray oscillogram, Fig. 57) with the result that the average current flowing through the grid leak increases very rapidly, which in turn results in a marked depression of the average grid potential.

The increased electron current to the grid as E_g tends to become

greater than $E_{p\,min}$ also represents a greater dissipation of energy upon the grid which is equivalent to an increase in the effective resistance R.

It is quite possible that any one of these three factors, in the absence of the others, would be sufficient to limit the oscillations; but as each springs into importance when the amplitude of the oscillation has reached about the same value, we shall not discuss the exact combination of the three which actually determines the amplitude.

Adopting the relation $E_{g\,max} = E_{p\,min}$ and making the additional assumption that the dynamic characteristic is straight (which is very nearly true) an expression for the amplitude of oscillation can readily

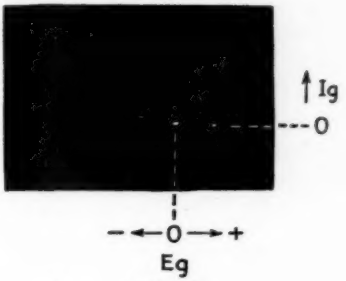

FIG. 57

be obtained. Introducing certain approximations this expression may be written,

$$i_{p1}(R+R_0) = \frac{1}{\sqrt{2}}\,\bar{E}_p, \tag{14}$$

in which terms of the order $\frac{1}{\mu}$ are neglected in comparison to unity. i_{p1} is the fundamental component of the space current, E_p is the mean plate voltage and the term R_0 is largely determined by the tube, and while not generally equal to r_p, is apparently not much different from it. This equation indicates that the amplitude of oscillation is practically independent of the value of μ.

The remarkably simple relation given by Equation 14 has been tested for a wide variety of circuits and tubes and has been found to hold with a very fair degree of accuracy. It may safely be taken as indicating quite approximately what response may be expected from any tube and circuit when operated at a particular applied E_p. The equation is likely to be more closely followed the more carefully the adjustment of circuit for maximum efficiency has been made.

The condition $E_{g\,max} = E_{p\,min}$ should not be considered as invariable. Adjustments can readily be made for which $E_{g\,max}$ will either be appreciably less or greater than $E_{p\,min}$. It is generally found however that these adjustments do not give the most efficient operation.

Additional information as to the relation between the amplitude of oscillation and certain circuit constants are given in Figs. 58, 59, 60,

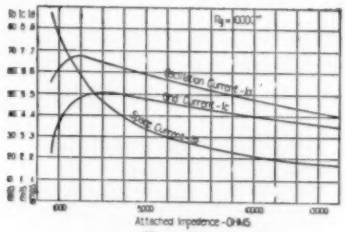

FIG. 58

61. Fig. 59 shows that an oscillator tube may present a well-defined condition of temperature saturation. Figs. 60 and 61 show that the

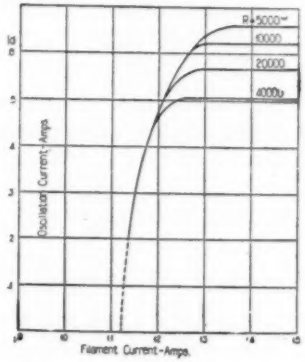

FIG. 59

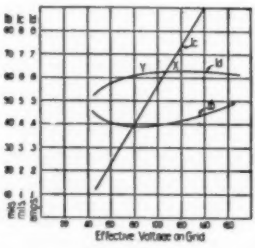

FIG. 60

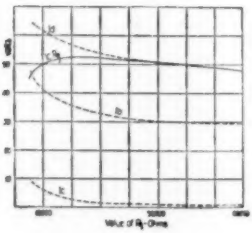

FIG. 61

value of the feed-back voltage and grid leak resistance r_g may be varied within wide limits without affecting the output markedly.

49. *Efficiency.* The efficiency of an oscillator may be defined as the ratio between the energy of oscillatory current and the d.c. energy supplied to the plate circuit. This leaves out of account the energy required to actuate the filament. The efficiency of oscillator circuits ranges all the way from a few per cent. to as high as 90% or better. The principal factors determining the efficiency are those which determine the amount of energy dissipated upon the plate of the tube. Inspection of the oscillogram, Fig. 55, shows that the sharper and narrower the plate current wave and the more nearly the plate voltage approaches zero, the higher will be the efficiency. In an extreme case such as the hypothetical one illustrated in Fig. 62 it is evident that the efficiency would be very large indeed. The μ of the tube

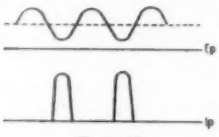

FIG. 62

largely determines the sharpness of the plate current wave and experience shows that for efficiences of about 50% and better, μ should be at least ten; an increase above this value does not result in any very large improvement. It is also generally true that for the highest efficiencies the circuit constants should be so arranged that R is at least four or five times as great as r_p, and it may advantageously be made 10 to 15 times as great. In these latter cases R_0 is relatively negligible compared to R in Equation 14 with the result that i_{p1} can be quite accurately calculated although the exact value of R_0 may be unknown.

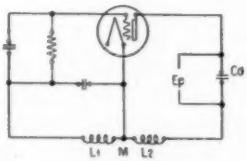

FIG. 63

50. *Types of Oscillating Circuits.* There are many different types of oscillating circuits and as they do not lend themselves readily to classification, only a few of the more common types will be described.

One of the simplest oscillating circuits is that shown in Fig. 63 which is characterized by a tuned grid circuit inductively coupled to a coil in the output. This type is satisfactory for low plate voltages and small powers, but is not to be recommended where large amounts of power and high efficiences are desired.

The condenser C_0 is inserted to prevent short circuiting of the plate battery or generator and may be made so large as to have no effect on the frequency.

A circuit of similar properties is shown in Fig. 64, the output being tuned instead of the input.

51. *Colpitts and Hartley Circuits.* Two very similar types of circuits which have proved satisfactory for a wide range of frequencies, voltages

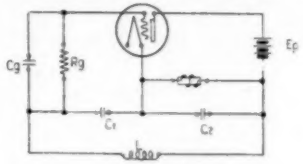

FIG. 64

and powers, and which yield very high efficiencies, are shown in Figs. 64 and 65, the former being known as the Colpitts and the latter as the Hartley circuit. In both circuits, as illustrated, the mean grid potential is secured by a grid leak. The blocking condenser C_g should

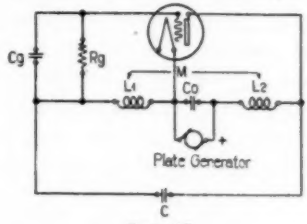

FIG. 65

be large enough to offer very little impedance to the flow of the alternating current which causes the variation of the grid potential, and as seen in Fig. 61 the resistance of the leak can be varied within wide limits without an appreciable effect upon the performance of the circuit. The source of plate potential may be either battery or generator. The choke in the generator supply should be sufficiently large to prevent an appreciable amount of oscillating current flowing through the generator. In order to avoid short circuiting the generator in the Hartley circuit, a large condenser C_0 is interposed as shown in

Fig. 65. For high frequencies it is customary to make this condenser so large that it has very little effect upon the frequency of oscillation.

The power of the oscillating current may be removed either by capacity or magnetic coupling, or it may be dissipated by resistance within the oscillating circuit. Magnetic coupling is very satisfactory for radio frequencies, but for the audio frequencies unless iron core transformers are used, capacity coupling usually proves more convenient. In regard to the use of iron core inductance and transformers, the reader should note the following section.

52. *Frequency of Oscillation.* It is frequently found that the tube causes an appreciable deviation from the calculated frequency of oscillation. This is not so much because the tube influences the normal operation of the oscillating circuit as it is due to the fact that the impedances of inductance coils (particularly when iron cored) and of the condensers are liable to vary in value with the current amplitude. Hence any change in applied plate potential, filament current, or output resistance, because of its effect upon the amplitude of the oscillating current, will give rise to changes in frequency. In designing circuits to operate at a constant frequency regardless of slight changes in tube constants, plate potential, filament current, etc., the most important requirement is to provide inductances and condensers whose impedances are independent of the currents they carry, and whose resistance components are very small.

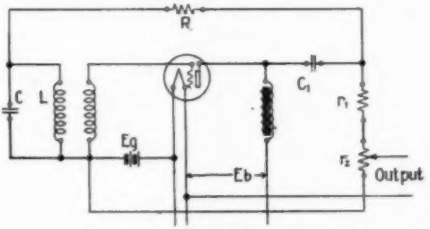

FIG. 66

53. *Oscillator for A.C. Measurement Purposes.* For many a.c. measuring purposes an oscillator whose output is both free from harmonics and constant in frequency is desirable. Such a circuit is shown in Fig. 66, the design of which is radically different from the oscillator circuits already discussed. It possesses a tuned input *LC* and coupling is supplied by the resistance *R*. *R* is usually given a value between 100,000 and 400,000 ohms. A negative grid battery fixes the average grid voltage, the emf of this battery being about

8 to 10 volts. It is customary to make the resistances r_1 and r_2 several thousands ohms apiece, r_1 being perhaps 5 times r_2. The condenser C_1 is merely a blocking condenser and should offer little impedance to the a.c.

54. *Range of Frequencies Obtainable with Vacuum Tube Oscillators.* Circuits have been constructed whose frequency is but a fraction of a cycle per second. The requirements of such a circuit are large inductance and capacity and very close coupling between input and output. A satisfactory circuit for low frequencies is that shown in Fig. 67; the two inductances taking the form of an iron core transformer.

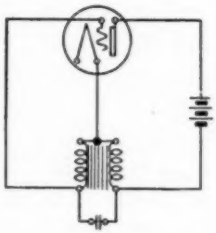

FIG. 67

At the other extreme, frequencies as high as 5×10^7 cycles per second can be obtained by means of tuned vacuum tube circuits of the Hartley or Colpitt type. At this point the coupling reactance of the tube becomes appreciable with that of the circuit.

Circuits capable of considerably higher frequencies have been described by Van der Pol,[38] Southworth,[39] Gutton and Touly,[40] and Holborn.[41] In all of these cases the oscillatory circuit is made up of distributed inductance and capacity connected to the tube in such a way as to utilize the capacity between the elements of the tube as a means of coupling.

The circuit shown in Fig. 68, when properly arranged, is as efficient as those used for lower frequencies and will give frequencies as high as 3×10^8 cycles per second. The oscillatory circuit is indicated by the heavy lines. It consists of a rectangle whose dimensions are appreciable with the wave length. Therefore, waves produced by variations in the electron emission through the grid are guided along the rectangle and are reflected at the ends. The reflected waves

[38] B. van der Pol, Phil. Mag., *38*, July, 1919.
[39] G. C. Southworth, Radio Rev., *1*, Sept., 1920.
[40] Gutton and Touly, Comptes Rendus, *168*, Feb. 3, 1919.
[41] F. Holborn, ZS. für Physik, *6*, p. 328.

produce the proper voltage changes on the grid to sustain oscillations. The ground imposed by the power leads places at least one point of the circuit at earth potential. If the condenser C is properly adjusted relative to the capacity between the grid and the plate the wave front can be made essentially perpendicular to the sides of the rectangle. It has been found that a large part of the power loss in the circuit is due to radiation. This circuit has been used as a basis of directive radio in which a metallic mirror was used to reflect the transmitted signals.

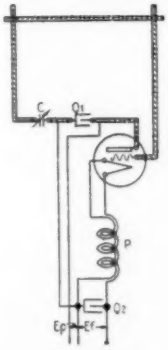

FIG. 68

Very different means of producing high frequencies have been used by R. Whiddington,[42] Barkhausen and Kunz,[43] and by Gill and Morrell.[44] In some cases they employ tubes having considerable residual gas. The frequencies produced depend on the relative voltages applied to the grid and plate. Probably the best explanation of this phenomenon has been given by Gill and Morrell. Frequencies higher than 3×10^8 cycles per second have been reported.

The most accurate way of measuring these high frequencies is by observing the length of standing waves produced on a parallel wire system. The constancy of the vacuum tube generator, compared with spark oscillators, combined with the fact that the sharpness of resonance in a parallel wire circuit is comparable with that in ordinary radio circuits, makes it especially adaptable to measurement purposes. It may be used, for example, to measure small inductances and capacities or to determine the dielectric constant of liquids.

[42] R. Widdington, Radio Rev., *1*, Nov., 1919.
[43] Barkhausen and Kunz, Phys. ZS., Jan. 1, 1920.
[44] Gill and Morrell, Phil. Mag., *44*, July, 1922.

Many of the corrections necessary when a damped source is used are eliminated.

55. *The Mechanically Coupled Oscillator.* In addition to the types of oscillators described above where the frequency is determined by inductance and capacity, we may have oscillators in which the frequency is governed by a mechanical system such as a pendulum or a tuning fork.[45] An example is shown in Fig. 68a. The two coils $L_p L_g$ are inserted in the plate and grid circuits of the tube. Variations

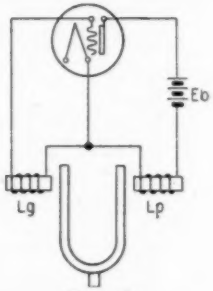

FIG. 68a

in the plate current through the coil L_p impress forces on the tuning fork which result in its motion. This motion of the fork causes variations in the magnetic field through L_g and induces a varying voltage on the grid. With the proper coupling, sustained oscillations result having a period very nearly that of the tuning fork.

The electrically driven tuning fork described above constitutes a very satisfactory source of either sound or electromotive force. Horton, Ricker and Morrison[46] have made improvements which make it constant in amplitude and frequency to six parts in a million over very long periods of time.

An entirely different form of mechanical coupling has been used by Cady.[47] The circuit is shown in Fig. 68b. It makes use of the piezoelectric effect and mechanical vibrations of a crystal. Variations in the plate current in the tube 9 cause a voltage change across the resistance 5. This is communicated to a crystal AB such as quartz by means of the plates 2 and 4. A transverse electric field applied to

[46] See Eckhardt, Karcher and Keiser, J. O. S. A. & R. S. I., *6*, p. 948, 1922; Eccles & Jordan, Phys. Soc. Proc. 31, Aug., 1919 and Phys. Soc. Proc. *32*, Aug., 1920; Abraham & Bloch, J. d. Physique, *9*, July, 1920.

[45] Journal of the A. I. E. E., 1923.

[47] Cady, Proceedings of I. R. E., *10*, No. 2; April, 1922.

such a crystal causes a change in its length. If this electric field be periodic, compression waves will travel along the crystal with a velocity depending on its density and elastic properties. These waves will, in turn, cause a varying electric field between plates 1 and 3 which may be communicated to the grid of the tube 7, amplified by 8, and finally transmitted to tube 9. This provides conditions for sustained oscillations having a frequency which is roughly inversely proportional to the length of the crystal.

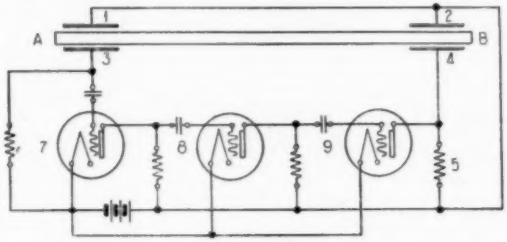

FIG. 68b

Cady describes oscillators ranging in frequency from 3×10^4 to 10^6, and states that the frequency is constant to about one part in 10,000. The effect of temperature change is not great.

X. MISCELLANEOUS APPLICATIONS OF THERMIONIC VACUUM TUBES

56. *The Tube as a Voltmeter.* The three-element tube may be used for the measurement of either d.c or a.c. voltages. In the case of d.c. voltages it is customary to apply the unknown voltage to the plate, counter-balancing this voltage with a known negative potential applied to the grid. Given the μ of the tube, it is then possible to calculate with a fair degree of accuracy the plate potential. The usual procedure is to adjust the negative grid potential to such a point that the plate current just becomes zero. The tube when used in this manner becomes an electrostatic voltmeter, and it is evident that to give accurate readings the tube should have a well-defined cutoff (see Sec. 8, Fig. 9).

In a somewhat similar fashion a.c. peak voltages may readily be compared with known d.c. voltages. A typical circuit is shown in Fig. 69. In operation a fixed plate voltage is applied to the voltmeter tube and a steady negative d.c. voltage is applied to the grid which is just sufficient to reduce the plate current to zero. The a.c. voltage is then superimposed in the grid circuit with the result that current

flows during the positive halves of the wave. If now the steady grid potential is made more negative until the plate current again just ceases to flow, it is apparent that this change in the steady potential just equals the peak value of the a.c. voltage.

For the measurement of very high voltages a special tube of the design shown in Fig. 70 will be found desirable, the grid being in the form of a screen which surrounds the filament. Such a tube may have a μ as high as 200.

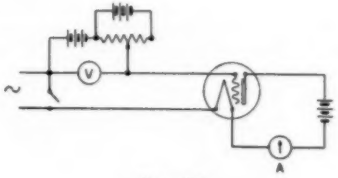

FIG. 69

A circuit similar to Fig. 69 may be so employed that the a.c. voltage to be measured causes a change in the space current meter reading, the negative grid potential being preferably so set that the conditions discussed in Sec. 16 are satisfied. The tube, due to its curved characteristic, acts as a detector and as pointed out in Sec. 27, the change in space current is approximately proportional to the square of the a.c. input voltage. For accurate work the circuit requires calibration but the calibration will in general remain good over long periods of time. This method is particularly useful for small voltages.

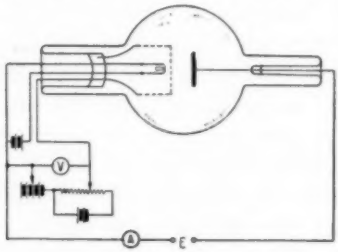

FIG. 70

57. *Power-Limiting Devices.* As pointed out in Sec. 4 the total emission from the filament at a given temperature is fairly sharply defined regardless of the plate voltage, so long as this exceeds the value required to give voltage saturation. The fact that the total emission is limited by the temperature may be used to control the

maximum current in a circuit. As an illustration, Fig. 71 shows its application to an alternating current circuit, the performance of which

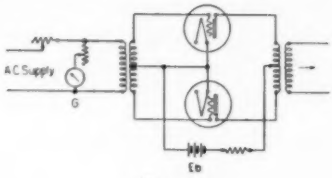

FIG. 71

is illustrated in Fig. 72. The introduction of such a device into an a.c. circuit will, of course, result in the generation of harmonics and may therefore, be objectionable.

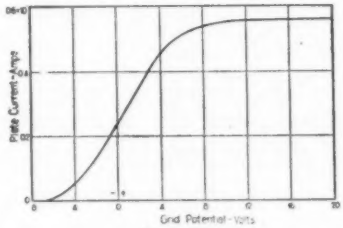

FIG. 72

There is almost no limit to the number of regulatory circuits which can be devised to employ the three-electrode tube.

58. *Voltage and Current Regulation of Generators.* The two-electrode tube with tungsten filament has been used to great advantage as a voltage regulator for a special airplane generator designed to de-

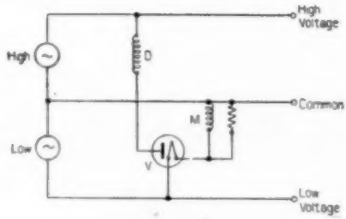

FIG. 73

liver both 28 volts and 300 volts. The circuit arrangement is illustrated in Fig. 73 in which *M* represents the main field winding, and *D* the differential winding which opposes *M*. The high voltage given by the generator when applied to the plate of the valve is sufficient to

produce a condition of voltage saturation. As the speed of the generator increases, the current through the main field winding and the filament increases, thereby giving rise to greater emission from the filament and a larger current through the differential winding. It was found possible to so design the valve as to yield the very close regulation illustrated in Fig. 74.[48]

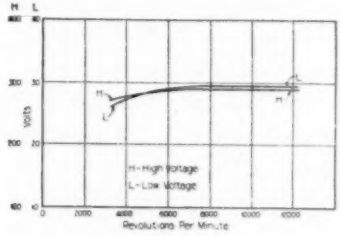

FIG. 74

The three-electrode tube can also be used as a voltage regulator for a generator as shown in Fig. 75. It is apparent that an increase in the voltage across the line tends to increase the current through

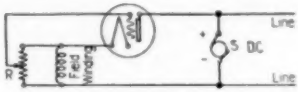

FIG. 75

the tube and resistance *R*. This in turn lowers the grid potential and tends to prevent an increase in current through the field winding.

The circuit shown in Fig. 76 illustrates an arrangement for maintaining a constant current from a generator. The operation of the device is apparent.

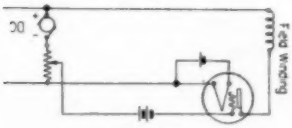

FIG. 76

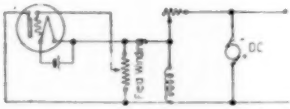

FIG. 77

Fig. 77 shows another arrangement for maintaining a constant generator voltage. In this circuit an increase in voltage tends to make the grid less negative, thereby reducing the resistance shunted across the field winding.

[48] Radio Telephony, by Craft & Colpitts, Trans. A. I. E. E., *38*, p. 330; 1919.

A somewhat similar arrangement can readily be applied to regulate the voltage delivered by a battery. Fig. 78 illustrates such a circuit. An increase in E_1 raises the grid potential, thereby increasing the

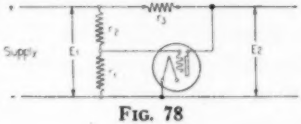

FIG. 78

current through the tube and the resistance r_3. By a choice of regulating tube and resistances such that

$$r_3 = \frac{r_1 + r_2}{r_1} \frac{dE_g}{dI_p},$$

it may readily be shown that the voltage E_2 remains constant. Since the regulation effected by this circuit is independent of frequency it may also be applied to a generator supply for elimination of commutator noise as well as voltage fluctuations due to changes in speed.

59. *The Ionization Manometer.* When gas is present in a three-electrode tube in quantities not sufficient to seriously affect the activity of the filament, and the plate voltage exceeds a value sufficient to produce ionization by collision, it has been found that the number of ions produced is proportional both to the pressure of the gas and to the electron current passing through the gas to the anode.[49] If now a small negative potential be applied to the grid, a certain fraction of the positive ions will be drawn to it and their number can be accurately measured by the current flowing in the grid circuit. The best arrangement is to apply the positive potential, not to the plate in the usual fashion, but to the grid, and apply the negative potential to the plate making it the collector of the positive ions. Dimensions of a satisfactory tube are given in Fig. 79. The values $E_g = 110$ volts and $E_p = -2$ volts have been found to give very satisfactory results, the electron current being .02 ampere, and K being equal to 0.10 for nitrogen and having approximately this value for air. The gauge equation may be put in the form,

$$P = K \frac{I+}{I-},$$

in which P is the pressure, K a constant depending upon the design of the tube, $I+$ the positive ion current, and $I-$ the electron current.

[49] O. E. Buckley, Procd. Nat. Acad., *2*, p. 683, 1916.

As it is necessary to know the value of $I-$, and since the emission from the filament is liable to vary somewhat with the kind of gas and its pressure, it will be found advantageous, if many readings are to be made, to place in the $I-$ circuit, the coils of a relay which is adjusted to close at a definite value of $I-$, and which, when closed, cuts in a shunt around the filament which will reduce its heating current. Automatic regulators of this type have been used with complete success. Experiment shows that the value of K remains constant for pressures as high as 1.5×10^{-3} mm. of Hg. and the lower limit is deter-

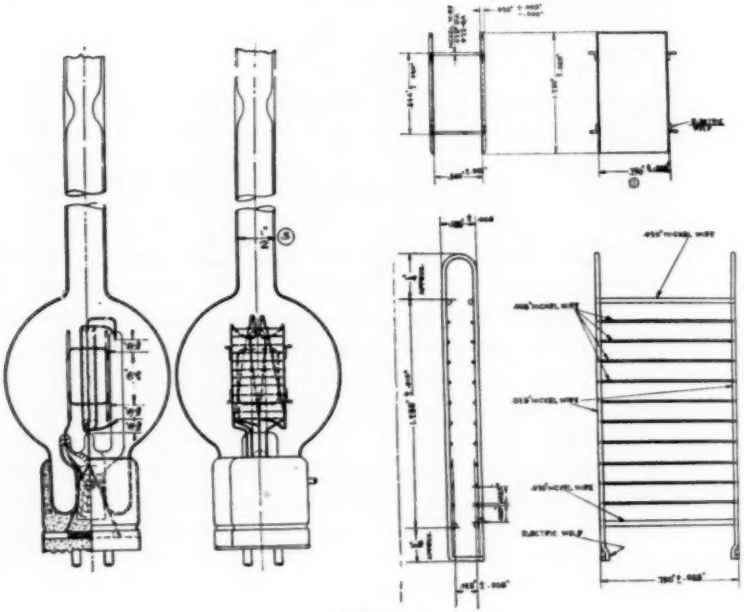

FIG. 79

mined very largely by the sensitivity of the current reading instruments. It follows that an ionization gauge can be calibrated by comparison with a McLeod gauge and, in use, extrapolated to low pressures.[50]

60. *Heterodyne Method of Generating Currents of Very Low Frequencies.* By impressing upon the grid of a detector tube two frequencies which differ by a very small amount (e.g., 99 and 100 cycles),

[50] Dushman, Phys. Rev., Oct., 1920, p. 854.

it is possible to obtain from the output of the detector the difference frequency of one cycle per second. This low frequency may be readily separated by means of a filter. It is apparent, however, that to maintain this difference frequency constant requires that the input frequencies be held within a very narrow range of variation.

61. *Thermionic Valve as a High Tension Switch.* If the plate circuit of a valve is inserted in a high tension circuit, the flow of current in the circuit may readily be stopped by cutting off the filament heating supply, thus making unnecessary the breaking of any contacts in the high tension circuit. In case the transmission of current in both directions is necessary, two valves may be used.

62. *Devices Employing Secondary Emission.* As pointed out in Sec. 9, the grid current in a three-element tube shows a negative resistance characteristic for a certain range of voltage, and various uses of this fact have been pointed out.[51]

63. *Electron Tube Oscillograph.* A special type of thermionic tube designed for oscillographic uses is of great importance as a laboratory instrument. These tubes, using the hot filament as a source of electrons, have certain marked advantages over the Braun tube with its gaseous discharge.[52] One of the very successful thermionic oscillographs has the following properties: anode potential 300-400 volts, sharp focus of electron beam, sensitivity of 1 mm. per volt between deflection plates and 1 mm. per ampere-turn when using magnetic deflection. Photographic recording is possible with relatively short exposures by using suitable fluorescent material.

AMERICAN TELEPHONE AND TELEGRAPH CO.
195 BROADWAY, NEW YORK CITY.

[51] See footnote 21.

[52] See J. B. Johnson, J. O. S. A. & R. S. I., Sept., 1922, or Bell System Technical Journal, Nov., 1922.

Observations on Fluorescence.—Perrin observes that the amount of fluorescence light emitted from an infinitely thick dilute solution of a fluorescent compound varies very rapidly with the concentration; in the ratio 130:1 in one instance where the concentration was changed in the ratio 1:20. An "infinitely thick" layer is a layer so thick that an increase of thickness produces no appreciable increase in the amount of light emitted. If the amount of light generated, the fraction of the incident light absorbed, and the fraction of the fluorescence light absorbed were all proportional to the concentration, then the amount of fluorescence light emitted by such a layer would be independent of concentration. As the two latter assumptions are justified and the conclusion is wrong, the amount of fluorescence light generated per molecule must decrease as the molecules are brought closer together. Yet compounds are often fluorescent in the solid state, which in the liquid state, or in solutions, in which the molecules are much further apart, do not fluoresce. Instances of this are given; for example, anthracene and phenanthrene. Solutions of such compounds as aesculin and sodium fluoresceinate, too highly concentrated to be fluorescent in the liquid state, fluoresce brightly when solidified in liquid air. The solids are fluorescent whether they are crystalline or amorphous. Perrin suggests that the fluorescence of the liquids is suppressed because of the very close encounters between molecules which occur during their wanderings, while in solids this factor is absent. [J. Perrin (Paris) C. R. *177*, pp. 469-475; 1923.]

K. K. DARROW

Optical Axes of Field Glasses.—Raibaud has described the elements of a device for measuring the departures from parallelism in the two emergent axes of a field glass and has discussed the methods for its adjustment.

Various arrangements of apparatus are possible. A pair of parallel collimators and a pair of parallel observing telescopes have been often used, the glass under test being placed between these with eyepieces toward the observing telescopes. Instead of the two parallel observing telescopes, a camera may be substituted having a lens sufficiently large to take in the beams from both eyepieces of the field glass. Another possibility is to use one large collimator and one large observing telescope, but in the estimation of the author this is less practical than a pair of parallel collimators and a large projection lens adjacent to the eyepieces and projecting the central part of both fields superposed on a screen. It is claimed that a simple lens is sufficiently good for the purpose. The characteristics of this type of device are considered in detail. The use of a mirror to bring the screen to a convenient position with respect to the binocular holder is recommended. [Jules Raibaud, Revue d'Optique, *1*, pp. 481-98; 1922.]

G. W. MOFFITT

THE ELECTROMETRIC METHOD OF MEASURING ACIDITY AND ALKALINITY

By W. T. Bovie

If we arrange similar metallic electrodes in contact with solutions of their salts as shown in Fig. 1, then the potential difference between the two electrodes is dependent upon the concentration of the positive (metallic) ions in the solutions which bathe the electrodes and upon the valence of these ions, but is independent of the nature of the metal and of the negative ions contained in the solution. If we neglect the

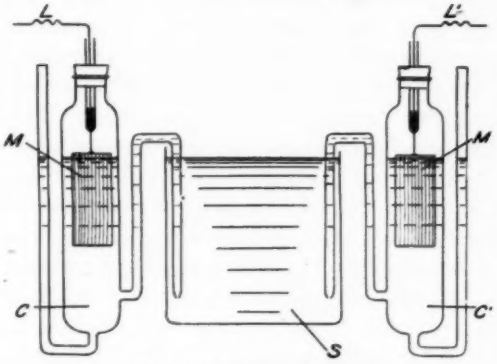

Fig. 1. *A Concentration Cell.*

M and M are metal electrodes.
C and C' are solutions of the salts of the metal M having a concentration of respectively C and C'.
S is a connecting vessel containing a solution of either C or C' or another electrolyte such as KCl.
L and L' are copper leads to the measuring instrument.

electromotive forces originating at the points of contact between the solutions the potential difference is given by the equation:

$$P = \frac{2 \cdot 303RT^{\circ}}{VF} \cdot \log_{10} \frac{C}{C'}$$

in which

P = potential difference in volts between the electrodes
R = the gas constant (8.315)
T = the absolute temperature.
F = the number of electrostatic units carried by 1 mol of monovalent ions (96,540)
V = valence of the metallic ions.

C and C' = concentrations of the metallic ions in the two solutions.

Knowing the concentration of the metallic ions in one of the solutions we may by the use of such a "concentration cell" determine the concentration of the metallic ions in the other solution, for we may measure the difference in potential between the two metal electrodes and substitute this value and the value of the ionic concentration of the known solution into the formula given above. Then since all of the terms in the equation excepting the one indicating the concentration of the unknown solution are given the equation may be solved for this unknown term.

It is usually possible to find the concentration of a salt solution if we know the concentration of its ions. If the solution is very dilute the salt will be completely dissociated and its concentration will be equal to that of the metallic ions. For more concentrated solutions in which the salt is not completely dissociated the concentration may be calculated providing the dissociation follows the Ostwald dilution law and it may be estimated to an approximate degree of accuracy for solutions in which the dissociation deviates from this law.

The value of this electrometric method of measuring concentrations lies in the fact that potential differences may be measured with considerable precision and as will be seen from an inspection of the formula given above the potential difference is proportional to the difference of the logarithms of the concentrations and therefore, the limits of error inherent in the method are proportional to the concentrations themselves. Thus the electrometric method enables us to measure both concentrated and dilute solutions with the same precision. This is not true of gravimetric methods. Solutions so dilute that a gravimetric analysis would give concentrations practically equal to zero can be readily measured by the electrometric method.

The method has its widest application in the measurement of the concentration of hydrogen ions in acid and alkaline solutions. It has been especially useful in those measurements of acidity and alkalinity where the presence of coloring matters obscure the end point, or where for other reasons the ordinary color-indicator methods are inadequate. It is the object of this article to set forth briefly the technic of the measurement of hydrogen ion concentration by the electrometric method.

The article, which has been written at the suggestion of the Editors of this Journal, does not profess to set forth new material nor has the author intended to write an exhaustive and detailed treatise upon

the electrometric method of measuring hydrogen ion concentrations. He will have accomplished his purpose if he acquaints a number of readers with this very interesting application of physical methods to chemical and biological problems and he will be amply repaid if some of the teachers of physics are induced to include in their physics courses a few laboratory exercises illustrating the electrometric methods of measuring ionic concentrations. He feels sure that anyone having charge of instruction in physics would gladly make such an inclusion if he could be made to realize how extremely useful a thorough discussion of the physical principles involved would be to those of his students who later concentrate in the fields of chemistry, biology or medicine. Many of the principles involved have a distinctive cultural value in relation to pure physics. Those who wish to pursue the subject farther will find an excellent discussion in Clark's book "The Determination of Hydrogen Ions" (Williams and Wilkins Co., 1922) in Michaelis' "Die Wasserstoffionenkonzentration" (Julius Springer, 1922) and Le Blanc's "Text Book of Electro-Chemistry" (The Macmillan Company, 1920); and in the numerous articles in the recent and current issues of the regular chemical and biological journals.

For the sake of completeness it seems advisable to state very briefly a few well known principles upon which the method rests.

Water dissociates into hydrogen ions H^+ and hydroxyl ions OH^- according to the Mass Law as follows:

$$[H^+]\times[OH^-] = \text{a constant} \times [H_2O]$$

in which the brackets indicate concentrations in mols per liter. The amount of dissociation is so small that it does not appreciably change the concentration of the water and it follows, therefore, that the product of this constant times the concentration of the water is also practically constant. This product is the so called "dissociation constant" of water and is usually represented by the symbol K_w. Thus:

$$[H^+]\times[OH^-] = K_w$$

The numerical value of K_w is not altered appreciably by dissolved substances provided their concentration is not great enough to materially affect the concentration of the water. At 22°C the value is 1×10^{-14}. It is not independent of temperature. In pure water the concentration of the H^+ must be equal to the concentration of the OH^- but when an acid is added to water the concentration of the H^+ is increased and since K_w is a constant the concentration of the OH^- must be decreased. When, on the other hand, an alkali is added to water the concentration of the H^+ is decreased. Hence, by measur-

ing the concentration of the hydrogen ions alone we may find the concentration of solutions of both acids and alkalies.

It is only necessary to bear in mind the relations between the concentration of the OH^- and H^+ expressed by the formula given above. This relation is shown graphically in Fig. 2, in which the logarithms of the concentrations of the H^+ are measured along the ordinate axis and those of the OH^- along the abscissa axis. In this figure the origin is placed in the upper right hand corner of the graph because the concentrations plotted are less than unity and their logarithms are negative numbers.

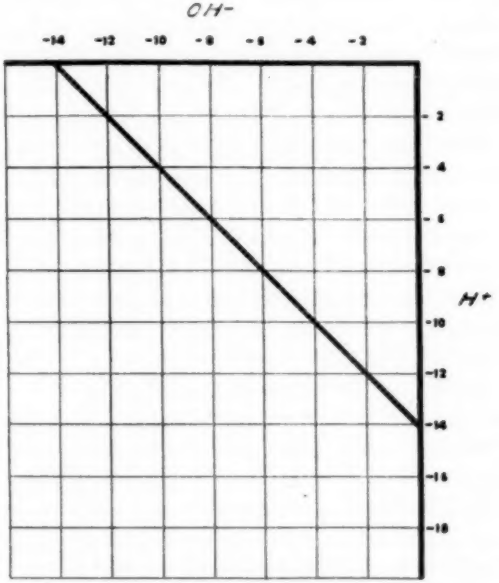

FIG. 2. *A graphical solution of the equation*
$$\log_{10}H^+ + \log_{10}OH^- = -14$$
The logarithms of OH^- are measured on the abscissa axis.
The logarithms of H^+ are measured on the ordinate axis.

Since we cannot form an electrode out of hydrogen alone, we use for the electrometric determination of hydrogen ion concentrations, an electrode (usually platinum or gold coated with platinum or palladium black) in which hydrogen is dissolved to a saturated equilibrium with hydrogen gas at atmospheric pressure. The earlier investigators arranged their hydrogen ion concentration cells as shown in Fig. 3.

In this arrangement one of the "hydrogen electrodes" is bathed by a solution having a known hydrogen ion concentration and serves as a "reference electrode." The other hydrogen electrode is bathed by the solution having the unknown hydrogen ion concentration. Electrical connection between the two solutions is made through an inverted U-tube filled with a solution of some electrolyte. In order that the diffusion potentials which occur at the interface between the connecting

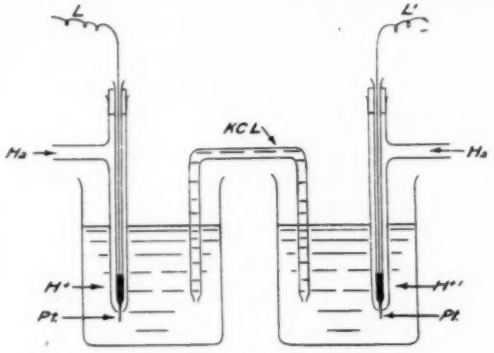

FIG. 3. *A Hydrogen Ion Concentration Cell.*
L and L' are the copper leads to the measuring instrument.
H_2 is the inflowing stream of hydrogen.
H^+ is the reference solution.
$H^{+\prime}$ is the solution of which the hydrogen ion concentration is to be measured.
Pt and Pt are platinum electrodes coated with platinum black.
KCl is a connecting solution of potassium chloride.

solution and the solutions bathing the electrodes shall be at a minimum it is advantageous to use for this purpose an electrolyte the ions of which have nearly equal transference numbers. A solution of potassium chloride is most frequently used.

Since the accuracy of the determination is dependent upon the constancy of the "reference electrode" any variation in the concentration of the solution bathing this electrode, any variation in the pressure of the hydrogen or any contamination of the hydrogen with which the electrode is saturated will vitiate the results. It is a common practice therefore to substitute for the hydrogen electrode a more suitable reference electrode due allowance being made for differences in potential between the substituted electrode and a normal hydrogen electrode. The one usually used is the reversible mercury-calomel-KCl-chain. A common arrangement is shown in Figure 4. Many

different types of hydrogen ion concentration cells, modifications of the one shown in Fig. 4, are described in the literature.

There is another type of hydrogen ion concentration cell which is not as well known and which has not been used as frequently as its merits warrant. In this cell the usual mercury-calomel-KCl-chain serves as a reference electrode but a glass electrode which is described later is immersed in the unknown solution in place of the electrode of platinum saturated with hydrogen gas. Within certain ranges of

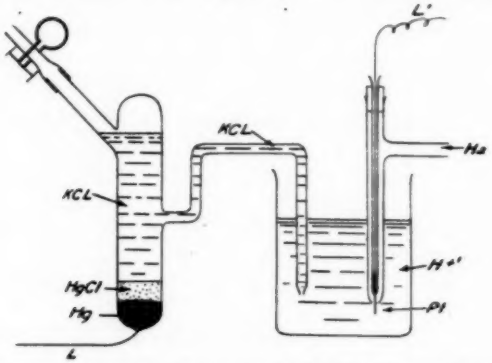

FIG. 4. *A Hydrogen Ion Concentration Cell With a Calomel Electrode.*
L and L' are the copper leads to the measuring instrument.
H_2 is the inflowing stream of hydrogen.
H^{+}' is the solution the hydrogen ion concentration of which is to be measured.
Pt is a platinum electrode coated with platinum black.
KCl is a connecting solution of potassium chloride.
HgCl is a paste of mercury and calomel.
Hg is mercury.

hydrogen ion concentrations the potential developed is defined by the law expressed in the equation given on page 149, that is, it is a function of the logarithm of the hydrogen ion concentration of the solution which bathes the glass electrode. There are, however, limitations as to the kind of glass which one may use. (Concerning the influence of the chemical composition of the glass and the effect of ions other than hydrogen ions contained in the solution, see Horovitz, Karl. Zeit. f. Physik. *15:* 369-398. 1923—Der Ionenaustausch an Dielektrikum I. Die Elektrodenfunktion der Gläser). No supply of hydrogen gas is needed. According to Haber a small amount of water is dissolved or occluded in glass either partly or entirely in a dissociated condition. The glass electrode, therefore, always contains a small

but constant concentration of hydrogen ions and at the interface between the glass and the solution there is a potential difference which is dependent upon the hydrogen ion concentration of the bathing solution.

The glass electrode is particularly valuable for measuring the concentration of hydrogen ions in the presence of strong oxidizing agents, and in the presence of certain substances which "poison" the platinum-hydrogen electrode: that is, substances which appear to displace the hydrogen dissolved in or absorbed upon the surface of the platinum. It is a matter of experience that the potential of the glass electrode is not affected by many of these substances.

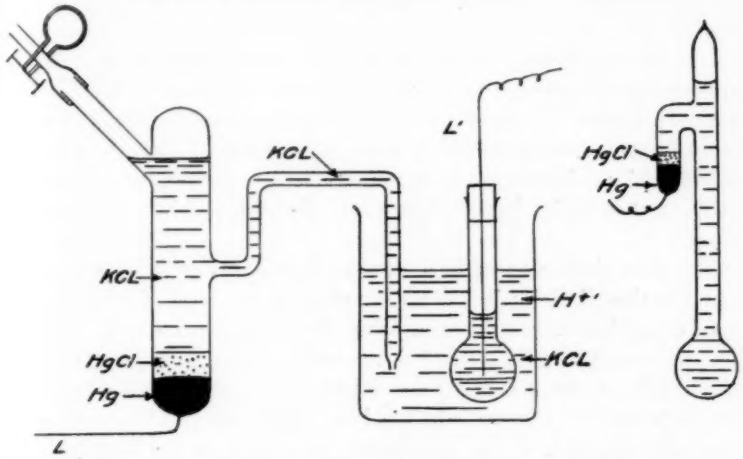

FIG. 5. *Left: A hydrogen-ion concentration cell with a glass electrode.*
Right: A Hughes glass electrode.
L and L' are the copper leads to the measuring instrument.
H^{+} is the solution the hydrogen-ion concentration of which is to be measured.
KCl is a connecting solution of potassium chloride.
HgCl is a paste of mercury and calomel.
Hg is mercury.

A hydrogen ion concentration cell with a glass electrode is shown in Fig. 5. The glass electrode may be formed by blowing a thin walled bulb on the end of a glass tube or one may use a thin walled boiling flask of 10 to 25 cc capacity. Because of the high electrical resistance of glass the wall of the bulb should not be over 0.5 mm thick.

The glass electrode may be filled with a solution of an electrolyte such as potassium or sodium chloride or it may be found more con-

venient to use a solution having a higher hydrogen ion concentration than that of any solution which is to be measured for then the potential difference across the interface between the inside solution and the glass bulb will be such that the polarity of the concentration cell will not be reversed within the range of hydrogen ion concentrations to be measured and it will not be necessary to include a pole changing switch in the electrical circuits of the measuring device.

Electrical connection with the solution inside the bulb may be made by a platinum wire or it may be found more convenient to use the electrode designed by Hughes and shown at the right in Fig. 5. It will be seen that in this electrode the "lead-in" wire is connected with the solution contained within the bulb through a mercury-calomel chain.

The glass bulb may be done away with entirely and the glass beaker containing the solution to be measured may be made to serve as the glass electrode. The beaker is supported on a small puddle of mercury spread on an amalgamated surface of a sheet of metal. The lead marked L' is soldered to the metal support. It is, of course, necessary to use a thin walled beaker, because of the high electrical resistance of glass.

The glass electrode may be used as a reference electrode in the manner recently described by Bovie and Hughes (Jour. Amer. Chem. Society —in press) but its use for measuring the actual hydrogen ion concentration of solutions is somewhat limited because of the fact that the magnitude of the potentials developed at the interface between the electrode and the solution which is being measured is influenced by the physical condition of the surface of the glass and the hydrogen ion concentrations cannot be calculated from the potential readings without knowing the previous history of the electrode. Its greatest usefulness will be found in electrometric titration and in the measurement of changes in the hydrogen ion concentrations of solutions.

Another type of hydrogen ion cell is shown in Fig. 6. In this cell a non-conducting liquid such as xylol, immiscible with aqueous solutions is used in the place of a platinum electrode. The potential at the interface separating the immiscible liquids is a function of the logarithm at the hydrogen ion concentration of the aqueous solution. Electrical connections with the oil layer are made through a metal plate. The reproducibility of the potentials which may be obtained with this type of concentration cell has not as yet been thoroughly investigated.

As with the glass electrode a supply of hydrogen gas is not necessary with the immiscible liquid type of electrode.

It will be seen from the above that the essentials of a hydrogen ion concentration cell are the following:

1. A "Reference Electrode" which is in chemical equilibrium with the solution which bathes it and which is reversible in the sense that the potential difference between it and the bathing solution is not altered by the passage of any electrical currents which must be taken from the cell in the process of making the potential measurements. A mercury-calomel chain is usually used.
2. Electrical connection between the solution which bathes the reference electrode and the solution of which we are measuring the hydrogen ion concentration. A glass tube filled with a solution of potassium chloride is usually employed.
3. A "Measuring electrode" from which hydrogen ions tend to pass into the solution being measured with a constant "solution pressure." This electrode must conduct the electrical current sufficiently to enable us to make potential measurements. Platinum, palladium, or iridium saturated with hydrogen gas, glass, or an immiscible liquid may be used.

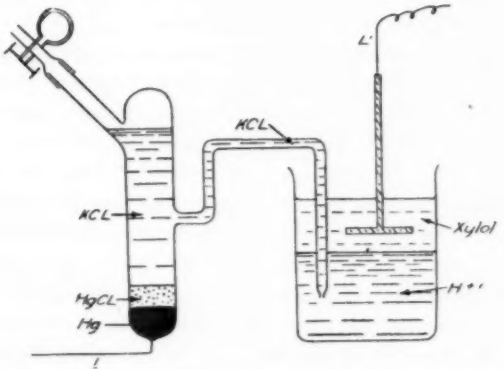

FIG. 6. *A hydrogen ion concentration cell with an immiscible liquid electrode.*
L and L' are the leads to the measuring instrument.
H^{+}' is the solution of the hydrogen ion the concentration of which is to be measured.
KCl is a connecting solution of potassium chloride.
HgCl is a paste of mercury and calomel.
Hg is mercury.

Now if these essentials are provided for, the hydrogen ion concentration cell may take on any form suitable for the particular conditions under which measurements are to be made. Descriptions of many modifications of the concentration cells illustrated in this article are to be found in the literature. For example, there is the concentration cell described by Keller (Jour. Ind. & Eng. Chem. *14*:395. 1922—"The Application of Ion Concentration Measurement to the Contro l

of Industrial Processes") for the continuous measurement of the hydrogen ion concentration of streams of flowing liquids; the concentration cell described by McClendon (Amer. Jour. of Physiol. *38*, p. 180; 1915) formed in the end of the stomach tube for the continuous measurements of variations in the hydrogen ion concentrations of the stomach and duodenal contents during the process of digestion; and the apparatus described by Schade *et al* (Zeit. für die Gesamte Experiment. Medizin. *24*, 11; 1921) for obtaining continuous records of the changes in the hydrogen ion concentration of the blood stream in which a small platinum-hydrogen electrode is fixed inside a glass hypodermic needle so as to be bathed by the flowing blood stream when the needle is forced into an artery or vein. The reference electrode in this concentration cell is the usual mercury-calomel-chain. Electrical connection between the reference electrode and the blood stream is made through a pad of absorbent cotton wet with a potassium chloride solution and applied to the skin at some convenient place.

Bacteriologists and others having to make measurements under special conditions will find modifications of the glass electrode useful. One might use a thin walled test tube containing the bacteriological culture solution for the measuring electrode, electrical connections with the reference electrode being made through the temperature controlling water bath to which an electrolyte has been added.

For the measurement of the difference in potential between the electrodes of a concentration cell, it is necessary to use a method which requires the passage of a minimal current through the cell. We cannot use a voltmeter for this purpose because of the large amount of current required to operate this instrument. A number of methods are available. We may, for example, use a quadrant electrometer. The instrument may, after proper calibration, be used as a direct reading instrument. It is more convenient, however, to use it as a null instrument in connection with a potentiometer as described later. A quadrant electrometer either as direct reading instrument or as a null instrument is to be recommended when concentration cells having a very high internal resistance are used; for example, when one is measuring the hydrogen ion concentration of conductivity-water or when glass or immiscible liquid electrodes are used, or when it is desirable to measure the hydrogen ion concentrations of tissue fluids in the living organism. A negligible amount of current passes through the concentration cell in charging the variable quadrants of the quadrant electrometer. A larger amount of current which leaks from the movable

vane on to the variable quadrants passes through the concentration cell. This leakage current may falsify the measurements if the internal resistance of the concentration cell is too high. For this reason the bulbs of glass electrodes should be made thin and the conductivity of immiscible electrodes should be made as high as possible. Insulation and electrical contacts require special care but once the apparatus is properly set up a quadrant electrometer gives very little trouble.

If a hydrogen ion concentration cell, consisting of two hydrogen electrodes be arranged as in Fig. 3, and the electrodes be connected through a sensitive high resistance galvanometer the galvanometer will show no deflection when there is no difference in potential between the electrodes, i.e., when the electrodes are bathed by solutions of equal hydrogen ion concentrations. If, therefore, we arrange a series of reference electrodes bathed with solutions having known hydrogen ion concentrations we may connect these in turn with the unknown solution until we find one which gives a zero deflection when the concentration cell is connected through the high resistance galvanometer and thus discover the hydrogen ion concentration of the unknown solution. While the method has but little to recommend it for most routine work, certain experimental conditions make it invaluable.

If the electrodes of the concentration cell be connected with a condenser the condenser will be charged to a potential equal to that of the concentration cell without the passage of a detrimental amount of current through the cell. After the connection with the concentration cell has been broken, the condenser may be discharged through a ballistic galvanometer and if the condenser used be of suitable capacity the potential difference may be determined from the magnitude of the galvanometer deflection. The ballistic galvanometer and condenser are calibrated against a standard cell.

Some form of potentiometer is, however, more frequently used. The principle is shown in Fig. 7. A source of emf e.g., a dry cell (A) supplies current to the circuit CDR. The potential drop in the portion CD is controlled by the rheostat R. The concentration cell forms a part of the shunt circuit CGH. H is a sliding contact which may be moved along CD, G is a null instrument such as a galvanometer or a quadrant electrometer which by its deflections indicates the direction of current flow in the shunt circuit. If the position of the contact H is such that the potential difference between H and C is just equal to and is in the same direction as the potential

difference between the electrodes of the concentration cell no current will flow through the shunt circuit and the null instrument will show no deflection. The procedure, therefore, is to adjust the contact *H* until the null instrument shows no deflection. The potential drop along *CH* is then equal to the potential of the concentration cell.

The potential drop along *CH* may be measured by a voltmeter connected between *C* and *H* at the time the adjustment of the contact *H* is made or, if the resistance throughout the circuit *CD* be known,

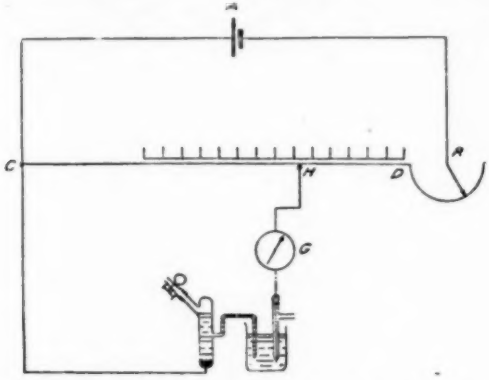

FIG. 7. *Simple potentiometer. A—the source of emf supplying current to the circuit ACDR; R—variable resistance; CGH—a shunt circuit containing the hydrogen ion concentration cell; G—a null instrument; H—a sliding contact*

a potential scale may be affixed to this circuit. A standard cell having a known emf, is substituted for the concentration cell, the contact *H* set to the scale reading of this known emf, and the rheostat *R* is adjusted until the null instrument shows no deflection. Potentials may then be read directly from the scale as long as the emf of the dry cell (*A*) does not change.

Instead of a potential scale one may apply a hydrogen ion concentration scale to the circuit *CD*, and the potentiometer is made direct reading and is therefore more convenient. McClendon (Am. J. Physiol. *38*, 186-190; 1915) and Bovie (J. Med. Research *33*, 295-322; 1915) have described direct reading potentiometers.

The equation given on page 149 states that the potential of the hydrogen ion concentration cell is dependent upon its absolute temperature and the direct reading potentiometers described by Bovie are provided with a device for making the proper temperature corrections through the use of an associated temperature scale.

The instrument described by Bovie is not only direct reading but is also direct recording. The hydrogen ion concentration scale in this instrument is not fixed under the circuit CD, but is laid off along the negative ordinate axis of a special sheet of Cartesian coordinate paper. The scale covers a range of hydrogen ion concentrations from normal acid to normal base and therefore consists of fourteen logarithmic units. Each logarithmic unit is ruled like the inverse scale of a slide rule. The entire scale from normal acid to normal base is like fourteen slide rule inverse-scales joined end to end. The entire hydrogen ion scale is shown in the figures below. A small section of the scale is shown in Fig. 8. A few examples will make the method of reading this scale clear. A reading at A is read 2×10^{-6}, at B, 6×10^{-6}, and at C, 10×10^{-6} or 1×10^{-5}, and at D, 4.5×10^{-6}. It will be seen that the reading is to

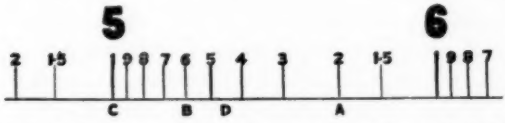

FIG. 8. *Hydrogen ion concentration scale. Description in the text.*

be associated each time with the negative exponent of ten to the right of it. A somewhat awkward but easily understood method of explaining the scale is as follows: the logarithmic reading is the numerator while the number to the right of the reading indicates the number of zeros to be written after one in the denominator of a common fraction which represents the hydrogen ion concentration of the solution in the usual terms of normality. 3.4×10^{-6} equals $\frac{3.4}{1{,}000{,}000}$ normal. If we wish to avoid the decimal point in the numerator we may write 34×10^{-7} or $\frac{34}{10{,}000{,}000}$ that is, the hydrogen ion concentration is thirty-four times as great as that of distilled water. The abscissa ruling of this coordinate paper is linear like that of ordinary cross section paper:—centimeter units divided into tenths. Since the ordinate units are negative and the abscissa units positive the rulings of the cross section paper lie in the lower right hand quadrant of the system of coordinates and the origin is at the upper left hand corner of the sheet.

The hydrogen ion concentrations are plotted directly upon this semi-logarithmic coordinate paper. The recording device is attached to the sliding contact H, and the coordinate paper is held on a flat

platen which slides under *CD*, Fig. 7. The coordinate paper is glazed with zinc oxide and the recording point of the printing mechanism is formed of a soft metal such as silver and is so designed that when it is pressed down on the paper and rotated, it draws a small circle with a black dot in the center. Because of the abrasive character of the zinc oxide glaze the soft metal marks like a lead pencil. No ink is necessary.

Now if the movements of the platen under *CD* are correlated with any independent variable of the investigation such as time, or the amount of solution added in electrometric titrations, etc., the instrument will plot a curve showing graphically the influence of the independent variable upon the hydrogen ion concentration of the solution. Changes in the independent variable are measured by the linear rulings in the direction of the abscissa axis and in the hydrogen ion concentration by the logarithmic rulings in the direction of the ordinate axis. This curve plotting feature of the potentiometer makes it especially valuable in biological investigations for the biologist is more often interested in how various factors change the hydrogen ion concentration of biological liquids than in either the total acid or alkali present or in the hydrogen ion concentration itself.

The curve plotting feature is also valuable in connection with acidimetry and alkalimetry for it enables the investigator to measure the amount of, and in many cases even to identify any acid or alkali which a solution contains, as surely as though an analytical analysis had been made.

The method of determining the amount of acid or alkali which a solution contains differs from the usual indicator method only in the fact that in this method we titrate to a particular hydrogen ion concentration at which the indicator changes color. In either method it is necessary to know the hydrogen ion concentration to which one must titrate in order to have the acid and alkali present in equivalent amounts (equivalent point). Obviously this is the hydrogen ion concentration of the salt formed by the titration. This concentration may be determined by measuring the hydrogen ion concentration of a solution of some of the pure salts made up to a concentration equal to that of the salt formed in the titration. There are, however, other methods of determining the equivalent point. The hydrogen ion concentration of the equivalent point may be calculated from the dissociation constants and the approximate concentrations of the acids and bases used in the titration. According to Hildebrand the

equivalent point is half way between the two sharp bends of the titration curve.

In regard to the precision of the electrometric method compared with the indicator method: the turning point of a good indicator under the best of conditions covers a range of from one to two of the logarithmic units of the hydrogen ion scale described above. These figures apply only to the range covered by the sharpest color change, the entire change covering a range of from two to three of these units. By ordinary methods, an expert with a good indicator can determine the hydrogen ion concentration to about 0.3 units. With the electrometric method one can readily measure the voltage to a precision of one millivolt. This means a determination of a hydrogen ion concentration to an accuracy of 0.017 of a logarithmic unit of the scale.

The titration curves of many physiological solutions, e.g., culture media do not have clearly defined end points and it becomes necessary to select arbitrarily an end point at a hydrogen ion concentration depending upon the nature of the determinations which are being made. Under these conditions the electrometric method of titration is more convenient than the indicator method for instead of being obliged to titrate to the end point of some indicator one can titrate to any desired hydrogen ion concentration.

A typical titration curve is shown in Fig. 9, Curve *A*. This is the titration of ten cubic centimeters of H_2SO_4 with 0.1 normal NaOH. The procedure was as follows: Ten cubic centimeters of the H_2SO_4 were placed in a hydrogen ion concentration cell such as is shown in Fig. 4. A stream of hydrogen was bubbled into the solution through the hydrogen electrode and when the cell had come into electrical equilibrium the hydrogen ion concentration at the abscissae reading *O*, was recorded on the record sheet. Then one cubic centimeter of 0.1 normal NaOH was run into the beaker from the measuring burette and when the concentration cell had again come into electrical equilibrium the second point on the curve at the abscissa reading was recorded. This process was repeated adding more or less of the NaOH solution as needed in order to determine the precise shape of the curve. One centimeter on the abscissa axis represents one cubic centimeter of NaOH added. As the NaOH is added the hydrogen ion concentration of the solution falls gradually until nine cubic centimeters have been added. The further addition of alkali causes the hydrogen ion concentration to decrease more rapidly. After ten and nine-tenths cubic centimeters have been added the curve falls precipitously from a

hydrogen ion concentration of 4.3×10^{-5} to 3.1×10^{-10}. Further addition of alkali produces a relatively small change in the hydrogen ion concentration of the solution. For an accuracy of one per cent the end point in this titration may be taken at practically any point along

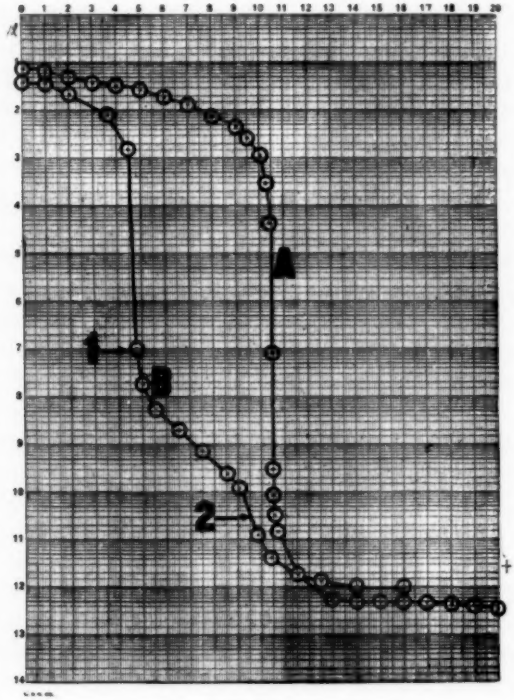

FIG. 9.

Curve A—The titration of 10 cc of N/10 sulphuric acid with N/10 sodium hydroxide.

Curve B—The titration of a mixture of hydrochloric acid and boracic acid with sodium hydroxide.

Arrow 1 indicates the equivalent point for the hydrochloric acid.

Arrow 2 indicates the equivalent point for the boracic acid.

The hydrogen ion concentrations are measured on the ordinate axis, the cubic centimeters of alkali added are measured on the abscissa axis.

the precipitous drop in the curve. Since we are titrating a strong acid with a strong base the salt formed is neutral in reaction and the equivalent point is at 1×10^{-7}.

Curve *B*, of Fig. 9, is the titration of a mixture of hydrochloric acid and boracic acid. The equivalent point for the hydrochloric acid

is at A, and for the boracic acid is at B. Ten cubic centimeters of the acid mixture were used in the titration. The alkali (NaOH) used was normal. The amount of these acids may be read from the abscissa ruling of the record sheet. This curve shows how it is possible to

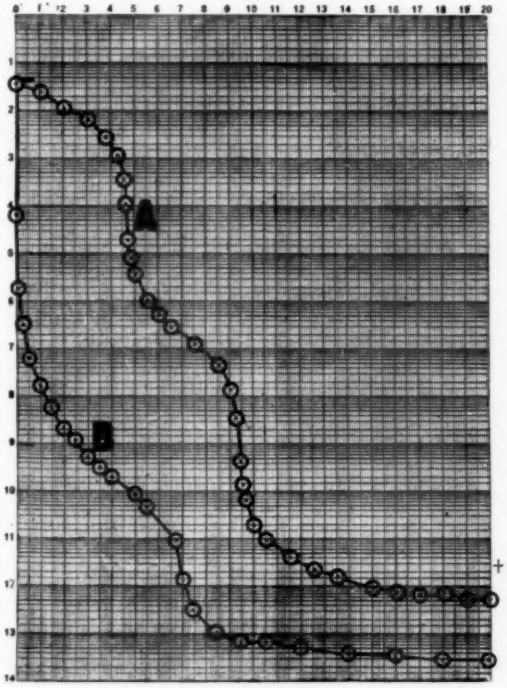

FIG. 10.

Curve A—The titration of a solution of ortho phosphoric acid with potassium hydroxide. In this curve we see two precipitous drops, the first one occurring when the primary sodium phosphate is formed, and the second one with the formation of the secondary sodium phosphate.

Curve B—The titration of a saturated solution of boracic acid with sodium hydroxide. The ordinate and abscissa scales are as in Fig. 12.

measure two acids in the same solution without making a chemical separation. The identity of the acids present is indicated by the shapes of the curves. The identification would be readily made by one familiar with the relation between the chemical constitution of acids and bases and the shapes of their titration curves.

Other titration curves are shown in Figs. 10, 11 and 12. The description of these curves will be found in the legends under the figures. It will be seen that the graphical representation of the progress of the titration makes it possible by a single titration to measure and identify one, or under certain conditions, several acids contained in a solution.

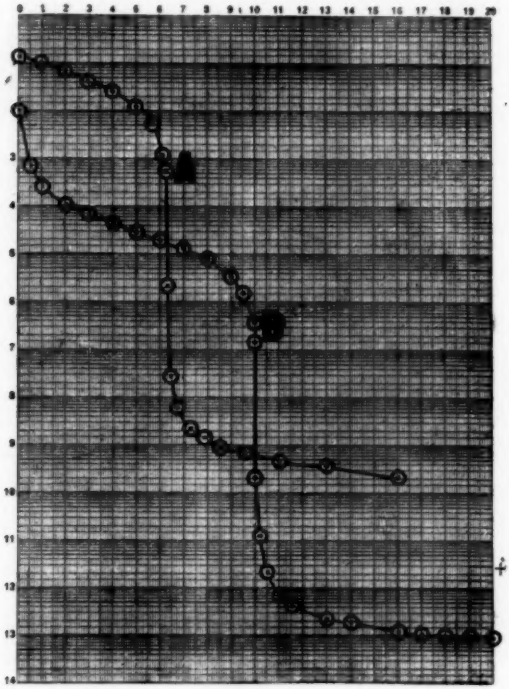

FIG. 11.

Curve A—The titration of a strong acid with a weak base: hydrochloric acid with ammonia.
Curve B—The titration of a weak acid with a strong base: acetic acid with sodium hydroxide.
The ordinate and abscissa scales are as in Fig. 12.

There are also certain conditions under which the investigator may thus determine the amount of a single acid or alkali in a solution in the presence of several others. Those acids or alkalies with which he is not concerned do not appear in the titration curve. The interpretation of such curves involves the solution of the rather complex algebraic expression. The author has developed a very simple method of solving

this expression graphically on the semi logarithmic paper described above. Details of the procedure will be described in a later publication.

There are many other problems which arise in connection with acidimetry and alkalimetry which the use of semi-logarithmic paper

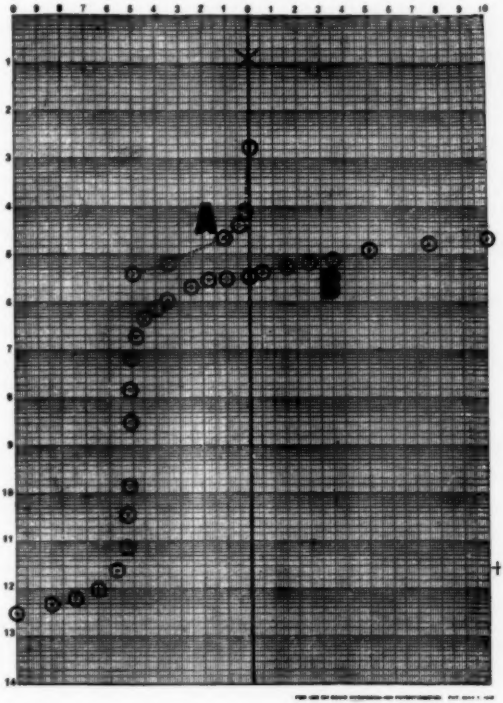

FIG. 12.

Curve A—The addition of crystals of sodium acetate to 0.1 normal acetic acid. The amounts of sodium acetate are indicated by the negative abscissa readings, five centimeters representing an amount of sodium acetate sufficient to make a normal solution of the salt.

Curve B—The titration of a mixture of acetic acid and sodium acetate. To the right with N/10 normal hydrochloric acid; to the left with N/10 normal sodium hydroxide. The curve shows the buffer action of a sodium acetate-acetic acid solution. The additions of alkali are represented as negative abscissa readings, the additions of acid are represented as positive abscissa readings (1cm=2cc). The hydrogen ion concentrations are measured on the ordinate axis.

The cross at 1.2×10^{-1} on the ordinate axis marks the total reaction of the acetic acid as determined by the titration with 0.1 normal NaOH. The actual reaction is 1.8×10^{-3}.

enables one to solve by graphical methods. These must also be left for a future communication.

An extension of the direct recording feature of the potentiometer described by Bovie has been made in automatic recording potentiometers which are used for the measurement of hydrogen ion concentration in the control of industrial processes.

BIOPHYSICS LABORATORY,
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AN ADJUSTABLE SCALE FOR ELECTRICAL INSTRUMENTS*

By Wm. F. Roeser

This scale was designed with the viewpoint of enabling an observer to read directly and correctly the measure of any physical phenomenon which is dependent upon the current through an electrical instrument and also with the idea of eliminating the time required to make corrections to the readings of instruments.

This scale consists of two parallel scales; one, rigid and fixed; and the other, elastic[1] and movable. The fixed scale is the same as on any electrical instrument and is graduated to show values of current or voltage, dependent upon the nature of the instrument. The elastic scale is molded from high grade rubber or it may be made from any elastic material. The graduations may be originally molded on the scale or printed on it to correspond to some general calibration. If for a millivoltmeter designed to be used with a thermocouple, the elastic scale will be graduated in either degrees Centigrade or Fahrenheit; if for an ammeter designed to be used with an optical pyrometer lamp, the elastic scale will also be graduated in degrees; and if for ordinary ammeters or voltmeters, it will be graduated the same as the fixed scale.

The two scales lie side by side with the pointer passing over both. The elastic scale is clamped to correspond to the calibration of the thermocouple, pyrometer lamp or the instrument itself. Two clamps, one at each end, are sufficient for the ordinary changes in calibration due to the deterioration of thermocouples and pyrometer lamps, but if the calibration changes considerably or a different thermocouple or pyrometer lamp with a different calibration is used with the original electrical instrument, it may be necessary to clamp the scale at one or more intermediate points. The intermediate clamps may either be small strips passing over the scale or sharp projections from the scale support into the back of the scale. For ordinary electrical instruments having an erratic calibration it may be necessary to clamp the elastic scale at every point at which the instrument was calibrated.

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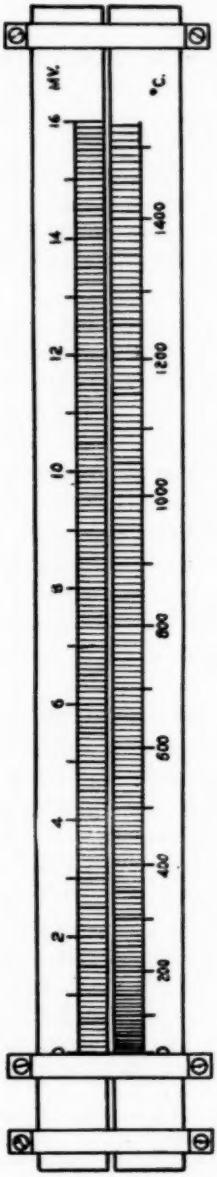
[1] In this article the term "elastic scale" is used to designate a scale made of a material which is highly extensible and contractible.



TOP SCALE IS PERMANENT AND FIXED TO DRUM.
BOTTOM SCALE IS ELASTIC AND HELD BY THE CLAMPS SHOWN.

On horizontal edgewise (wall type) instruments whose scale is cylindrical, the friction of the rubber against the drum is of considerable importance and errors due to this cause must be eliminated during the setting of the elastic scale. For instruments equipped with a flat circular scale, a slot of a depth equal to the thickness of the scale may be used to hold it in shape. In this case the scale should be thick and molded into a sector of a circle. A straight scale is satisfactory only for wall type instruments.

A white rubber scale graduated in degrees of temperature to correspond to an average thermocouple calibration was used very successfully with a wall type millivoltmeter. For changes in the temperature-emf relation of $\pm 5\%$ of full scale deflection, clamps as shown in the diagram were sufficient. For changes of $\pm 10\%$, one intermediate clamp was required but two were necessary to obtain an accuracy of 0.1% of full scale deflection.

The graduations were ruled on the scale with drawing ink and the scale could be read as easily and accurately as the ordinary scale. It was graduated for a normal position of tension (elongation about 12%) to make possible compensation by shortening without using compression.

It is a well established fact that rubber deteriorates, particularly at the surface, when exposed to light or heat. The deterioration is more rapid when the rubber is under tension. It is necessary to have the scale under tension for all positions because any attempt to compress the scale will cause it to buckle. Manufacturers have probably not used this type of scale because its life is much shorter than the life of the instrument. Although this is a disadvantage it is certain that good rubber will outlast several calibrations under normal usage.

After a mold is made or a printing device set up these scales can be replaced easily and cheaply.

BUREAU OF STANDARDS,
WASHINGTON, D. C.,
JULY 23, 1923.

The Ionization of Nitrogen by Electron Impact.—Continuation of Smyth's experiments on mercury vapor; electrons from a hot filament are accelerated by a voltage V_1 from the filament to the first grid, and retarded by a voltage V_2 from the first to the second grid, while the ions produced between first and second grid are drawn through the latter and accelerated by an additional voltage V_3 between grid and plate; there is a narrow slit in the plate, through which the ions pass with energies between V_3 and V_2+V_3, and enter a positive ray analysis chamber in which they are deflected by a strong magnetic field, making it possible to calculate the value of m/e. The chief difficulty lies in maintaining a sufficiently high pressure of nitrogen in the region between first and second grid and a sufficiently low pressure beyond the second grid; this is done, as far as possible, by feeding nitrogen in behind the first grid and pumping it rapidly out beyond the second grid. V_2 is equal to V_1 if V_1 is less than 40, and equal to 40 if V_1 is greater than 40 volts; in the latter case the electrons pass through the second grid, but are swept away by a magnetic field beyond it. At $V_1=100$ volts, the ions formed are N_2^+, N^{++} and N^+ (the last might be N_2^{++}, so far as the method can distinguish; but this is believed unlikely, because Thomson has found that multiply charged molecules rarely occur, and because it would be difficult to account for N^{++} without N^+). N_2^+ makes its appearance at $V_1=16$ volts, about; N^{++} at 24, and N^+, strangely, not till a higher value 28 (about). As the voltage V_1 is raised towards 600, N^+ suddenly becomes much more prominent and N_2^+ suddenly much less prominent at about 375 volts, which happens to be the K excitation potential of nitrogen. N_2^- was also observed, and an impurity for which $m/e=17.75$, probably OH_2^+. [H. D. Smyth (Cavendish) Proc. Roy. Soc., *A104*, pp. 121-134; 1923.]

KARL K. DARROW

Direction of Motion of Fast Secondary Electrons Produced By Polarized X-Rays.—The writer "using a beam of scattered x-rays about 90% polarized, has obtained stereoscopic photographs of β-ray ionization-tracks by the cloud method. These photographs show that most of the β-particles were ejected in a direction nearly parallel to that of the electric force of the polarized beam of x-rays. There is, however, a variation on either side of this direction. The photographs also support C. T. R. Wilson's conclusion that a large majority of the β-particles have a velocity component in the direction of propagation of the x-rays." [F. W. Bubb, (Washington University); Nature, *112*, p. 363; 1923.]

K. K. DARROW

APPARATUS FOR THE DETERMINATION OF COLOR IN TERMS OF DOMINANT WAVE-LENGTH, PURITY AND BRIGHTNESS*

By Irwin G. Priest

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ABSTRACT

Description of an assembly of apparatus consisting of a "monochromatic colorimeter" (previously so called) and complete accessory apparatus for the determination of dominant wave-length, purity and transmission or reflection for white light under definitely specified conditions.

* Partial report on cooperative investigations in colorimetry by the Bureau of Standards and the Munsell Research Laboratory, communicated to the Optical Society of America, Cleveland, Oct. 26, 1923.

Published by permission of the Director of the Bureau of Standards, U. S. Department of Commerce, and the Director of the Munsell Research Laboratory.

The apparatus is described as it has been actually used at the Bureau of Standards since March 1923.

Salient features of the apparatus and its use are:

(1) The use of a specified artificial white light, field illumination being from incandescent filament lamps.

(2) The diffusion sphere for the diffuse illumination of samples to be examined by reflection.

(3) The convenient arrangement for handling samples to be examined in either transmitted or reflected light.

(4) The convenient arrangement for the simultaneous adjustment of dominant wave-length, purity and brightness in the mixture field to match the juxtaposed sample field.

(5) The analysis of the mixture of homogeneous and white light by a substitution flicker method.

Details of design of the apparatus, explicit directions for its use and data on its performance as regards precision and accuracy, including extensive tests of the additivity of homogeneous and white brightnesses, are given. These data support the theorem that brightnesses due respectively to homogeneous and heterogeneous stimuli are strictly additive.

I. INTRODUCTION

It has long been a well known principle of physiologic optics that any color[1] (excepting the purples), due to whatever spectral distribution of radiant energy in the stimulus, can be matched by a stimulus consisting of a suitable mixture of (1) homogeneous energy and (2) heterogeneous energy which alone would evoke the hueless sensation known as gray.[2] To be specific, let us assume a photometric field in one half of which (known as the sample field) appears the color to be matched while the other half (known as the mixture field) is illuminated by a mixture of homogeneous light and "white light."[3]

Let this mixture be adjustable as regards:

(1) the wave-length of the homogeneous light,

(2) the component of field brightness due to the homogeneous light,

(3) the component of field brightness due to the heterogeneous light,

these three adjustments being independent.

[1] "Color" is understood as "sensation." See "Report of Colorimetry Committee," J. O. S. A. & R. S. I. *6*, pp. 531-534; 1922.

[2] Grassmann: Pogg. Ann., *89*, p. 70; May 1853, (or Grassmann's "Math. and Phy. Werke," *2*, part 2, p. 162). Popular expositions may be found in Rood's, "Modern Chromatics," Abney's "Colour Measurement and Mixture" and Luckiesh's "Color and its Applications."

[3] "White light" is not at present particularly well defined. Tentatively it may be regarded as heterogeneous light having the spectral distribution of sunlight or of a complete radiator ("black-body") at a temperature of about 5000° or 6000° absolute. Cf. H. E. Ives, Trans. I. E. S. *5*, pp. 196-198; 1910.

Priest, B. S. Sci. Pap., *417*.

Troland, J. O. S. A. & R. S. I. *6*, pp. 557-564, 1922.

By means of these three independent adjustments, let the mixture field be made to match the sample field as regards all three of the attributes of color, viz: ***brilliance, hue, and saturation.***[4] Then, by definition, we have:

(1) Dominant Wave-length,

Λ is the wave-length of the homogeneous light;

(2) Purity,

$$p \text{ is } \frac{B_\Lambda}{B_x};$$

(3) Impurity,

$$i \text{ is } \frac{B_w}{B_x};$$

where

B_Λ is the mixture field brightness due to the homogeneous energy alone,

B_w is the mixture field brightness due to the heterogeneous energy alone,

B_x is the sample field brightness.[5]

The brightness of the sample field is determined by the incident light and the transmission or reflection of the sample for this light.

So, if B_s be the brightness of the sample field when illuminated with white light (without the sample in place) and B_x be the brightness when a transparent sample of any color is interposed in the path of this same white light, the sample's

Transmission for white light,

$$T_w \text{ is } \frac{B_x}{B_s}.$$

Similarly, if B_s be the brightness of the sample field when illuminated by the reflected light from a standard non-selective diffusely reflecting surface which is illuminated by diffuse white light and B_x be the field brightness when a sample is substituted for the standard, the

[4] Cf. "Report of Colorimetry Committee," J. O. S. A. & R. S. I. *6*, pp. 534-535; 1922.

[5] These definitions should be supplemented by the following statements:—

(1) The values of Λ, p, and i as well as T_w or R_w found are subject to a "personal equation" of the observer.

(2) The determination of p (and consequently i) as well as R_w or T_w in general must be predicated upon an assumed method of measuring relative field brightness for colors of different quality.

With the first of these facts we shall not deal in the present paper. The second will be considered below.

sample's diffuse reflection for white light,

$$R_w \text{ is } \frac{B_z}{B_s}.$$

Assuming a standard observer, a standard white light and a standard method of measuring the relative field brightness for colors of different quality,[6] the specification of Λ, p (or i), and T_w (or R_w) constitutes a unique color specification.

Various suggestions and, in some cases, more or less perfected instruments designed for the determination of Λ, p (or i), and T_w (or R_w) have been proposed and developed by others.[7]

However, such instruments and methods as have been proposed and come into limited use have left much to be desired as regards:

(1) Definite provision for illumination of both the sample and the mixture field by standard white light;

(2) Definite provision for the diffuse illumination of samples to be measured by reflected light;

(3) The method of measuring the relative field brightness for colors of different quality;

(4) Complete adaptation to the convenient prosecution of routine determinations under standard conditions;

(5) Accuracy of measurement.

The purpose of the present installation at the Bureau of Standards has been to provide a complete unified assembly of colorimeter and accessory apparatus adapted to the convenient and accurate measurement of Λ, p, i, and T_w or R_w under definite specified conditions. The installation of this apparatus as described below was completed in March 1923. Since then it has been in almost constant use and, while not by any means to be regarded as perfect or final in its present form, it has been used with a high degree of satisfaction by the author and others. Inasmuch as considerable work has already been done with this apparatus and an intelligent consideration of the results must be predicated upon a knowledge of the apparatus, it has been decided to publish a description of it in its present state, although improvements

[6] Quality is that aspect of color determined by hue and saturation.

[7] Abney: "Colour Measurement and Mixture." London, 1891, particularly Chap. 13.

Nutting: "A new precision colorimeter," *B. S. Bull. 9*, pp. 1-5; 1913. U. S. Patent No. 1,026, 878; May 21, 1912.

Luckiesh: "Color and its Applications," 2nd Ed. 1921, pp. 99-101.

H. E. Ives: J. O. S. A. & R. S. I. 7, p. 291; 1923.

Adam Hilger, Ltd., London, Advertising Circular, entitled, "The Nutting Colorimeter."

are contemplated. The principal improvements contemplated are:—(1) more finished and better mechanical construction; (2) the use of a rotatory dispersion filter instead of blue glass to provide standard white light; (3) a higher sample illumination for dark samples.

It will be understood that this paper deals with details of instrument design and experimental technic rather than with anything fundamental or novel in principle. However, such details are of prime importance in the application of the fundamental principles to practical work in colorimetry.

II. SALIENT FEATURES OF METHOD AND APPARATUS

(*Due reference to Figs. 1, 2 and 3 will serve to render the following exposition of principles more concrete and definite.* A detailed description of these Figures is given in section III.)

The salient characteristic features of the apparatus and method as they have been used for several months may be briefly described as follows.

1. CENTRAL IDEA

A square photometric field, the edge of which subtends about 4° at the observer's eye, limited by a black diafram, is bisected by a vertical division line obtained by a silver-strip cemented photometric cube.[8] One half will be designated as the sample field; the other as the mixture field.

Both halves of this field may be illuminated by the standard white light.

The white light illumination of the mixture field arises by way of transmission through rough[9] ground glass. The white light illumination of the sample field arises by way of diffuse reflection from a standard white material.[10] As an initial adjustment care is taken to adjust the two illuminations so that the two halves may be matched with the standard white material in place.

The color of a sample to be examined by transmitted light is presented in the sample field by interposing the sample in the path of the

[8] Arrangements to use an illuminated surrounding field have also been made; but, to date, very little use has been made of it. It will probably prove a desirable feature. Larger field sizes are also possible in the present apparatus. The effect of selective reflection of the silver strip is eliminated by the experimental procedure (*vide infra*).

[9] The transmission of fine ground glass is selective with respect to wave-length.

[10] Pending further quantitative investigation of absolute selective reflection and the establishment of a better standard, a magnesium carbonate block which is *apparently* "white" is being used tentatively.

white light from the standard white material just before it enters the photometric cube. The color of a sample to be examined by reflected light is presented in the sample field by substituting the sample for the standard white material.

Homogeneous light may be added to the white light in the mixture field in such a way that the observer can adjust independently (1) the wave-length of the homogeneous light, (2) the component of field brightness due to homogeneous light, (3) the component of field brightness due to white light. He makes these adjustments so as to match the sample field.

2. STANDARD OF WHITE LIGHT

The present tentative *primary* standard of white light is that of a Planckian radiator at a temperature of 5000° K.

The present tentative *working* standard of white light is the light from a gas-filled tungsten lamp (referred to hereafter as the "white lamp") as modified by transmission through a suitable thickness of Ives-Brady[11] daylight glass.

The working standard is calibrated in terms of the primary standard by methods already described.[12]

This calibration takes the form of measurements of color temperature, as indicated by the light passing the blue filter and the ground glass (see 1 above), as a function of current in the lamp. Subsequently, the working standard is realized by setting the lamp current (from the calibration curve) to correspond to color temperature 5000° K (or other value, if desired).[13]

3. METHOD OF ILLUMINATING SAMPLES TO BE EXAMINED BY REFLECTED LIGHT

For the illumination of the sample (or the standard white material interchangeable with it) a hollow white lined sphere containing a small

[11] H. E. Ives: Jour. Frank. Inst., *177*, pp. 471-499; 1914.
E. J. Brady: Trans. I. E. S. *9*, pp. 937-952; 1914.
The author is greatly indebted to Mr. Norman Macbeth, President of the Macbeth Daylighting Co., for supplying this glass which could not be obtained commercially.

[12] Priest: "Colorimetry and Photometry of Daylight and Incandescent Illuminants," J. O. S. A. and R. S. I., Dec. 1923.

[13] It is planned to later use a rotatory dispersion filter directly in the colorimeter and so eliminate the use of the blue glass filters. Cf. Priest: Phy. Rev. (2) *11*, p. 502; 1918.
The fact that the spectral distribution of energy in the Ives-Brady or other blue filter white light is not a close approximation to a Planckian distribution, altho competent to evoke the same color, raises the question of the propriety of using it for a standard of general validity.

lamp (referred to hereafter as the "sample lamp") is used. The sample (or white standard) is brought into position from outside the sphere so as to close a small round hole in its wall. Diametrically opposite this hole is another hole through which the sample is viewed by way of the colorimeter telescope and the photometric cube. A line through the centers of these holes will be designated as the axis of the illumination sphere. The sample is illuminated only by light diffusely reflected from the sphere wall, being shaded from direct illumination from the lamp by a small opaque white screen.

4. METHOD OF OBTAINING THE HOMOGENEOUS ILLUMINATION IN THE MIXTURE FIELD

The homogeneous illumination in the mixture field is obtained by way of a direct-reading transmission grating spectrometer of the constant deviation type supplied by Adam Hilger.[14] This instrument has been provided with a pair of new bilateral slits, one for the collimator and one in the plane of the real spectrum (i.e. the focal plane of the telescope objective). A real image of the incandescent tungsten strip of a gas-filled tubular-bulb ribbon-filament lamp (referred to hereafter as the "spectral lamp") is projected on the collimator slit. As used for most observations to date, the collimator slit width has been 0.17 mm, giving a spectral range of one millimicron. The slit width in the real spectrum has been 0.85 mm, covering a wave-length range of five millimicrons in the spectrum.

5. ARRANGEMENT OF PARTS FOR MIXING THE HOMOGENEOUS AND WHITE LIGHTS, AND FOR PRODUCING AND VIEWING THE JUXTAPOSED MIXTURE AND SAMPLE FIELDS

The plane of the spectrometer collimator and telescope axes is horizontal. A vertical plane through the telescope axis contains the axis of the sample illumination sphere. This sphere lies wholly below the plane of the collimator and telescope. The axis of the illumination sphere is vertical, the hole where the white material standard is placed being at the bottom, so that the surface of the white material standard (or a sample to be examined by reflected light) is horizontal. (Cf. II-3 above). The axis of the illumination sphere intersects the telescope axis (projected) at a point between the spectrometer telescope objective and the total reflection prism which carries the diffraction grating. The silver strip photometric cube is

[14] Adam Hilger's catalog, Fig. D 25; March, 1921.

placed at this intersection so that the normal to the silver surface lies in the plane of the axes of the telescope and the illumination sphere and bisects the angle between them. Light from the white material standard (or a sample being examined by reflected light) is thus reflected into the spectrometer telescope by the silver surface. White light is superimposed on the homogeneous light from the grating by reflection at 45° from a transparent plano-parallel glass plate standing in a vertical plane between the photometric cube and the prism which carries the grating. The homogeneous light passes through this glass plate and then, together with the white light, through the photometric cube and into the spectrometer telescope. For viewing the field a small short focus astronomical telescope, known as the viewing telescope, is placed in the usual position of the ocular of the spectrometer telescope. Its axis is coincident with the axis of the spectrometer telescope, and its objective is placed as near as convenient to the slit which lies in the real spectrum. (Cf. II-4 above). The viewing telescope is focused on the edge of the silver strip in the photometric cube. A black slide in the focal plane of its objective carries field diaframs of various sizes. On looking into the viewing telescope the observer thus sees an illuminated field limited by the diafram in the focal plane of this telescope. This field is bisected by a vertical line which is the image of the edge of the silver in the photometric cube. One half of this field is illuminated by white light (see II-6 below) modified by reflection from or transmission through the sample as the case may be. The other half is illuminated by a mixture of homogeneous light and white light.

6. METHOD OF USING THE BLUE GLASS FILTERS TO REALIZE THE WORKING STANDARD OF WHITE LIGHT

One blue glass filter is placed between the ground glass (see II-1 above) and the glass plate which reflects the white light into the telescope objective (see II-5 above). Another covers the upper hole of the illumination sphere (see II-3 and 5 above).

The current in the white lamp having been set to its standard value (see II-2 above) and the material white standard being in place, the current in the sample lamp (see II-3 above) is adjusted[15] so that the two juxtaposed fields are matched in quality of color, a brilliance match

[15] Actually a number of such settings of current are made and the mean taken as the standard current for sample illumination.

being simultaneously obtained by adjusting the distance between the ground glass (see II-1 above) and the white lamp. (Cf. 7, just below).

7. METHOD OF ADJUSTING THE MIXTURE OF HOMOGENEOUS LIGHT AND WHITE LIGHT TO MATCH THE SAMPLE

The wave-length of the homogeneous light (λ) is adjusted by turning the wave-length drum of the spectrometer. The observer does this with his right hand. The homogeneous component of field brightness is adjusted by adjustment of current in the spectral lamp. The observer does this by means of a sliding contact tube rheostat controlled by his right foot. The white light component of brightness is adjusted by adjusting the distance of the white lamp from the ground glass (see II-1 above). The observer does this by pulling a cord with his left hand. With attention and practice the observer learns to make these three adjustments simultaneously with entire satisfaction.

8. METHOD OF DETERMINING PURITY AND IMPURITY BY ANALYSIS OF THE MIXTURE OF HOMOGENEOUS AND WHITE LIGHT

In the determination of purity or impurity, two distinct steps are involved, namely, (1) the *adjustment* of the mixture to match the sample, and (2) the *analysis* of this mixture after it has been made. The method of making the mixture has just been described.

The analysis is made by a *substitution flicker method*. A polished monel metal flicker disk of two open and two closed quadrants is placed so that its plane of rotation, which is vertical, intersects the projected axis of the spectrometer telescope at a point between the objective and the photometric cube, making an angle of about 45° with this axis. While the sample is being matched the flicker disk is set stationary with an open sector in the line of sight. It is therefore entirely out of consideration in that process.

When a closed polished sector is moved into intersection with the axis of the telescope, the polished surface reflects into the telescope light from a milk glass diffusion screen which is illuminated by diffuse transmission of light from a lamp on the side opposite the flicker disk. This lamp, known as the comparison lamp, is mounted on a photometer track so that its distance from the milk glass may be adjusted and read directly from a scale on the track. This arrangement is called the auxiliary photometer. The procedure for the analysis by which p and i are determined is now as follows:

(a) The mixture of homogeneous and white light having been adjusted so that the mixture field is matched with the sample field as

regards brilliance, hue, and saturation, a two-degree field diafram in the focal plane of the viewing telescope is slid into position so as to show the center of the mixture field while covering the sample field.

(b) The flicker disk being started and its speed adjusted to maximum sensibility, the distance of the comparison lamp from the milk glass is adjusted to give minimum flicker in the field. Call this distance D_{A+w}.

(c) The white light in the mixture field being cut off by a shutter, leaving only the homogeneous component, the distance of the comparison lamp from the milk glass is again adjusted to give minimum flicker. Call this distance D_A.

(d) The white light shutter being reopened so as to restore the white as before, and the homogeneous light being extinguished, the distance of the comparison lamp from the milk glass is again adjusted to give minimum flicker. Call this distance D_w.

(e) Then it is obvious that[16]

$$p = \left(\frac{D_{A+w}}{D_A}\right)^2$$

and

$$i = \left(\frac{D_{A+w}}{D_w}\right)^2.$$

9. METHOD OF DETERMINING THE TRANSMISSION OR REFLECTION OF A SAMPLE FOR WHITE LIGHT

In determining T_w or R_w we observe only the sample field limited by the two-degree field diafram with flickering illumination from the sample and the comparison lamp. The mixture field and the parts for its illumination are out of consideration . The procedure is as follows:

(a) The current in the sample lamp being set at its standard value for white light, and the white material standard being in place, the distance between the comparison lamp and the milk glass is adjusted to give minimum flicker in the field. Call this distance D_s.

(b) To measure T_w, insert the sample in the path of light between the illumination sphere and the photometric cube, and adjust the distance between the comparison lamp and the milk glass to give minimum flicker. Call this distance D_{xT}.

(c) To measure R_w, substitute the sample for the white material standard, and adjust the distance between the comparison lamp and the milk glass to give minimum flicker. Call this distance D_{xR}.

[16] (Cf. III-4 and V-4 below)

(d) Then it is obvious that

$$T_w = \left(\frac{D_s}{D_{xT}}\right)^2$$

and

$$R_w = \left(\frac{D_s}{D_{xR}}\right)^2.$$

III. DETAILS OF DESIGN

1. GENERAL ARRANGEMENTS OF PARTS

The general arrangement of the apparatus is shown in the accompanying figures, which, in connection with their legends and the exposition of general principles in the preceding section are, for the most part, self-explanatory.

Fig. 1 shows a horizontal section in the plane of the axes of the collimator and telescope. Fig. 2 shows a vertical section containing the axis of the telescope. The whole assembly is shown in perspective in Fig. 3. It occupies a specially built table about three meters long, fifty centimeters wide and ninety centimeters high, set in a small dark room devoted exclusively to this work.

The apparatus is suitably arranged for the conduct of measurements by two persons, the observer and an assistant. Without leaving the ocular the observer has command of the following adjustments: wave-length of homogeneous light (by right hand); current in spectral lamp (by right foot *or* right hand); distance of white lamp from ground glass (by left hand); distance of comparison lamp from milk glass (by right hand); flicker disk speed (by left hand); current in sample lamp (by right hand).[17] From his position, the assistant has control of the currents in the white lamp and the sample lamp. He can read the wave-length and the distance of the comparison lamp from the milk glass. He can also read the current in the spectral lamp as well as the currents which he regulates.

2. THE LAMPS, THEIR ELECTRIC CIRCUITS AND CURRENT REGULATION

There are four lamps serving to illuminate the field in various ways for various purposes. These are: the "sample lamp," in the illumination sphere; the "white lamp," which by way of a blue filter provides white illumination in the mixture field; the "spectral lamp" which, by way of the spectrometer, provides homogeneous illumination in

[17] The last named adjustment is used only in the initial adjustment of this current to its standard value. The others are all used in the ordinary course of measurement of Λ, ϕ and i.

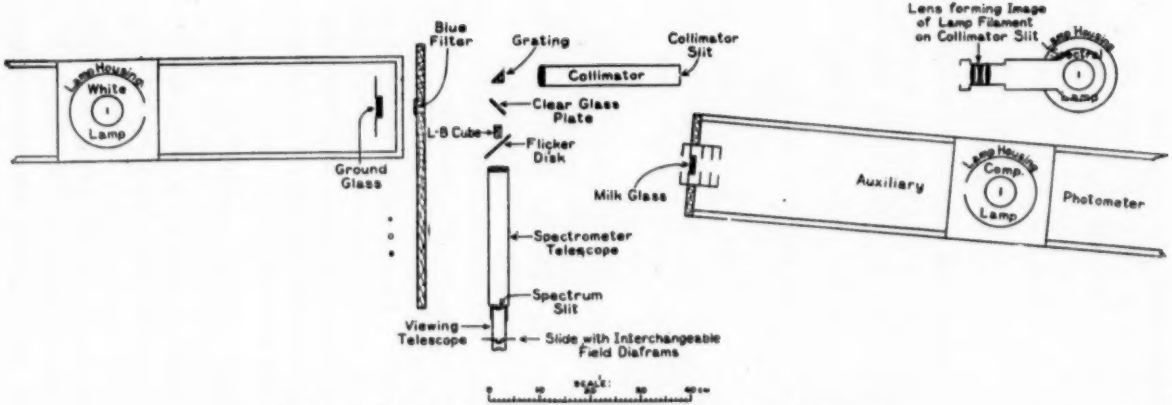

FIG. 1. *Horizontal Section through the Axes of the Telescope and Collimator.*

the mixture field; the "comparison lamp" which illuminates the milk glass used with the flicker disk. They are all gas-filled tungsten lamps. The sample lamp is an automobile headlamp rated at 9.0 v, 2.14 a. The spectral lamp is a ribbon-filament lamp rated at 8.2 v, 19 a. The white lamp and the comparison lamp are concentrated-filament (stereoptican) lamps rated at 400 w, about 110 v or 115 v. For the sake of constancy and long life, it has been arranged that all

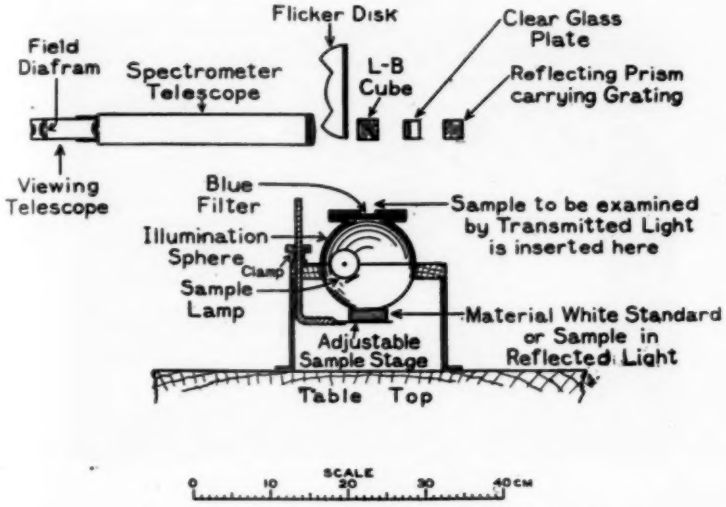

FIG. 2. *Vertical Section through the Axis of the Telescope.*

of the lamps are operated notably below these commercial ratings. The sample lamp was "seasoned" at 1.9 a, and its standard current in operation is 1.78 a. The white lamp was "seasoned" at 3.3 a, 119.5 v. Its standard current in operation is 3.04 a at 99.1 v. The spectral lamp is rarely if ever operated at over 18 a.

The requirements of accuracy to be met by current regulation are:—

(1) The reproduction from day to day of the working standard white light by the white lamp and the sample lamp; (2) the constancy of the white brightness and the homogeneous brightness relative to the comparison milk glass brightness during the few minutes occupied in measuring p and i; (3) the constancy of the sample illumination relative to the comparison lamp illumination during the few minutes occupied in measuring T_w or R_w.

The first requirement is easily met by ordinary ammeter regulation with sliding rheostats of the tubular type.

To meet the second and third requirements we depend upon the following provisions:

(a) All currents are supplied from storage batteries.

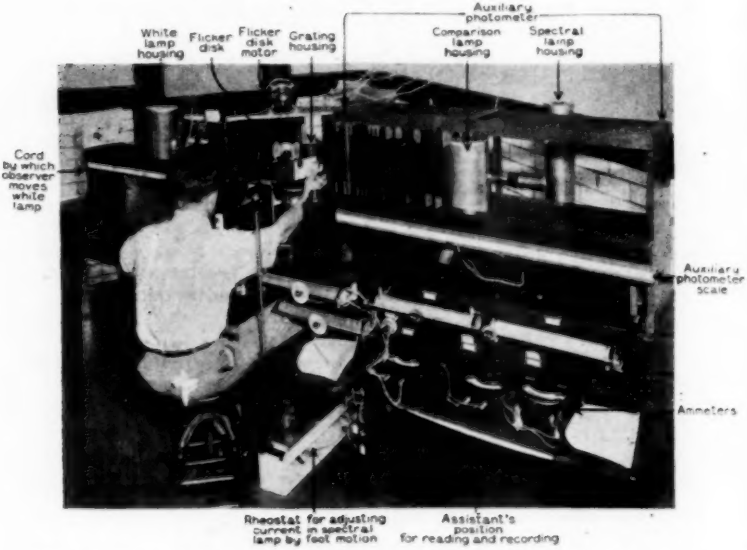

FIG. 3. *Perspective View of Whole Apparatus. The observer is shown in the process of adjusting the mixture of homogeneous light and white light to match the sample. His right hand is on the wave-length drum of the spectrometer. His left hand is on the cord by which he moves the white lamp. His right foot is on the slide by which he adjusts the current in the spectral lamp. In the flicker analysis his right hand would reach the cord under the auxiliary photometer track.*

The illumination sphere is partly shown to the left of and below the observer's right wrist. The spectrometer collimator is mostly hidden by the auxiliary photometer. The rheostats shown are for control of lamp currents. The rheostats for control of flicker speed are concealed by the observer's head. The small telescope in the foreground (over the observer's right shoulder) is used by the assistant in reading the wave-length scale.

(b) The white lamp and the comparison lamp are in series on the same circuit and the sample lamp is shunted around a series resistance in this same circuit.

(c) The spectral lamp being on a circuit entirely independent of the other lamps, care is taken to measure D_A *immediately* after D_{A+w}, postponement of the measurement of D_w for a few minutes being more

readily permissible because of the condition that the white lamp and the comparison lamp are in series on the same circuit.

The wiring diagram of the white lamp, the sample lamp, and the comparison lamp is shown in Fig. 4. The wiring diagram of the spectral lamp is shown in Fig. 5. The rheostats are all of the common tubular type.

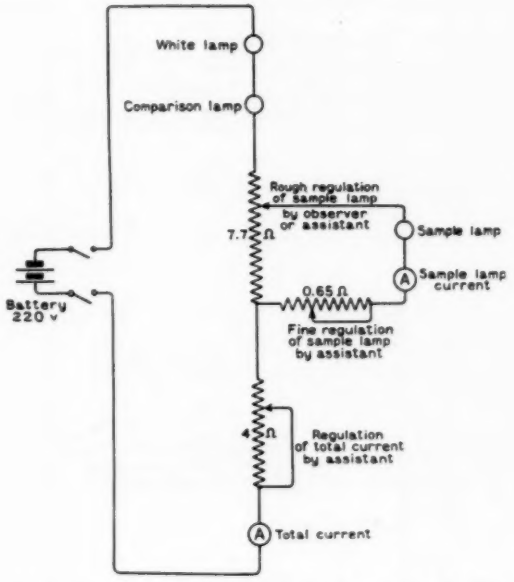

FIG. 4. *Wiring Diagram of White Lamp, Comparison Lamp and Sample Lamp.*

For the regulation of current in the spectral lamp by the observer's foot, the rheostat has been mounted on the floor beneath a track about 70 cm long, in front and a little to the right of the observer's stool. A sliding plate rides on this track. A slot in this plate engages the ordinary handle of the rheostat slider while adjustable clamps on top of the plate engage the sole of the observer's shoe. The adjustment is satisfactorily made by pushing the foot forward and backward.

3. THE FLICKER DISK AND ITS SPEED REGULATION

The flicker disk is of monel metal about 12 cm diameter and 3 mm thick. The surface is well polished, giving a good mirror. The open and closed sectors are cut to accurate quadrants with edges beveled

on the back. The disk is driven by belt from a motor which is mounted on a strong platform above the other apparatus. (See Fig. 3.)

It is a matter of prime importance to the sensibility of flicker photometry that the observer be able to regulate the speed of the flicker disk with a minimum lag between the adjustment made and the response to it in speed. The inertia of the rather massive metal disk is not favorable to this requirement. In spite of this, the requirements

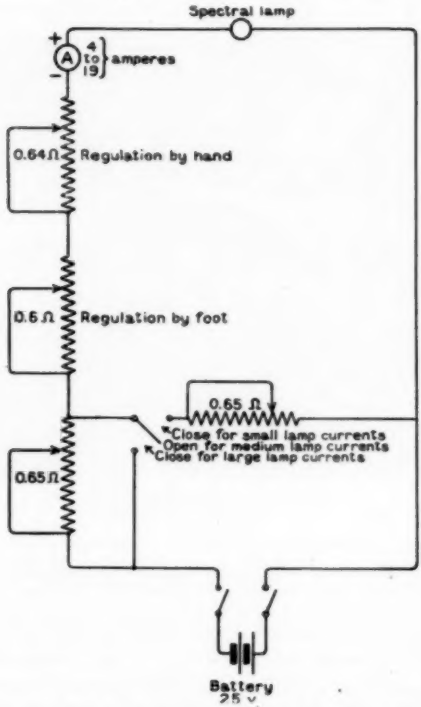

FIG. 5. *Wiring Diagram of the Spectral Lamp. The double throw single pole switch provides for three rough adjustments of current, the fine adjustment in each case being made by either the hand or foot rheostat.*

of speed regulation have been fully met in an entirely satisfactory way by the type of electric regulation adopted. A shunt motor of rather high power (1/12 hp) is used. The wiring diagram is shown in Fig. 6.[18]

[18] The author is indebted to Mr. H. B. Brooks and Mr. R. Davis for advice on this method of speed regulation.

4. SCALE EXTENSION FOR THE AUXILIARY PHOTOMETER

The distance between the comparison lamp and the milk glass illuminated by it may be varied between about 13 and 100 cm. In some measurements it is necessary to make the apparent brightness of the milk glass less than can be attained by this range of adjustment. This is accomplished by inserting between the milk glass and the

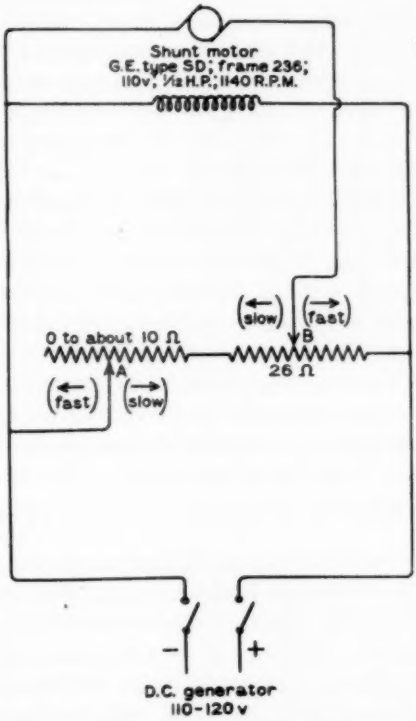

FIG. 6. *Wiring Diagram of the Flicker Disk Motor. The speed may be varied as indicated by movement of slider on either rheostat. The fine adjustment is normally made by movement o slider A after B has been set by trial at about the required speed.*

flicker disk a smoke glass of known transmission. This filter has been calibrated *in place* by use of the comparison lamp itself. With constant illumination from the sample lamp on the material white standard, the settings of the comparison lamp for minimum flicker (in the sample field with 2° diafram) were made with the filter and without the filter.

The transmission of the filter is then

where $\left(\frac{D_f}{D_0}\right)^2$

D_0 is the distance between the milk glass and the comparison lamp with the filter out; and

D_f is the distance between the milk glass and the comparison lamp with the filter in.

The transmission of the filter which we use has been found by this method to be 0.1258. This filter is permanently mounted in the apparatus so that it may be conveniently inserted or removed from the path of light between the milk glass and the flicker disk on instant notice. When it is used the factor 0.1258 is appropriately entered in the formulas for p, i, T_w or R_w.

In measuring very low values of i another procedure is possible, and indeed preferable, in that it enables the flicker measurements to be made at a higher field brightness, which is very desirable. When the white component in the mixture has been set for a very low value of i, the white lamp will be at a considerable distance from the ground glass screen. Instead of measuring this low brightness directly, we may first move the white lamp nearer the screen, and then, having made the flicker measurements at the resulting higher brightness, enter an appropriate factor in the formula for i. If the white brightness of the mixture field varied inversely as the square of the distance of the white lamp from the ground glass, this factor would be $\left(\frac{d}{D}\right)^2$ where D is the distance set in making the mixture and d is the smaller distance set to give a greater brightness. However, since the ground glass is not a *perfect* diffuser it will be necessary to make an empiric calibration instead of *assuming* the inverse square law in this case. In remodeling the apparatus it would be advisable to design it so as to satisfy the inverse square law. This will require a more nearly perfect diffusing screen, and this in turn demands higher intensity in the white lamp.

IV. Explicit Directions for Use of the Apparatus

1. Preliminary Adjustments

The collimator being removed from the spectrometer a positive ocular is focused on its slit[19] and this arrangement, used as an

[19] An adapter cap fitting over the slit is provided to hold the ocular.

astronomical telescope, is focused on a distant object by adjustment of the distance between the slit and collimator objective. This adjustment having been clamped, the ocular is removed and the collimator returned to the spectrometer.

A positive ocular is then focused on the spectrum slit and, the collimator slit being illuminated by a helium lamp (or other source of homogeneous light) and the grating set to make the wave-length scale reading approximately correct, the spectrometer telescope is focused on the spectral image of the collimator slit by adjustment of the distance between the spectrum slit and the telescope objective; and the spectrum slit is made parallel to the image of the collimator slit. This adjustment is then clamped.

All parts of the spectrometer, the glass plate, the photometric cube, the illumination sphere, and the viewing telescope are then adjusted and aligned to give a uniformly illuminated perfect photometric field (with heterogeneous illumination on the collimator slit).[20] The viewing telescope is then replaced by a positive ocular focused on the spectrum slit and, with helium illumination on the collimator slit, a wave-length calibration of the spectrometer is made, the readings corresponding to centering of the several homogeneous collimator slit images in the spectrum slit. A curve is then plotted showing corrections to be applied to scale readings to give true wave-lengths. In subsequent work from day to day or whenever any disturbance may be suspected, this correction curve is checked at one point, usually for wave-length 587.6 millimicrons.

The spectrometer being in adjustment with the viewing telescope in place and spectral lamp extinguished, the standard current for white is approximately set in the white lamp, and the observer increases the current in the sample lamp[21] to match quality of color, matching brilliance simultaneously by adjustment of the distance of the white lamp from its ground glass. An assistant then sets the current in the white lamp to its standard value and *maintains it constant* while the observer makes the precision match by adjustment of current in the sample lamp. This adjustment of current is made a number of times,

[20] In practice, it may be necessary to make the adjustments of the grating for approximate wave-length reading and the adjustments for the field illumination alternately until both are satisfactory.

[21] The shunt should always be set so that the sample lamp draws little or no current when the main circuit through the white lamp and the comparison lamp is closed. Neglect of this precaution may result in momentary overload and even burning out of the sample lamp.

say ten, and the average current is taken as standard thereafter. From time to time the color match should be checked with these currents at their standard values.

2. METHODS OF OBSERVATION

(*Cf. II-7, 8 and 9 above*).

The adjustment of the mixture of homogeneous and white light to match the sample requires the highest degree of ATTENTION on the part of the observer. In this respect this adjustment differs notably from the simple photometric practice of making a brilliance match between two fields of like quality. With little practice this simple photometric procedure may become almost automatic requiring little or no THOUGHT. On the other hand, in the adjustment of three variables to match the sample, the observer must continually THINK of what needs to be done. An aimless or absent-minded adjustment will be quite futile; and the observer who does not have in mind the correlation between the psychologic attributes of color, namely, hue, saturation and brilliance and the three stimulus adjustments, namely, wave-length, purity and brightness, will be entirely confused and helpless.

Since attention and alertness are prime requisites it is advisable to undertake such work in the morning if possible, and not to continue observations for more than an hour or two in one period. Likewise a comfortable posture and the general physical comfort of the observer are of great importance to the attainment of the highest precision.

The mean of a series of settings of either λ or p will probably most nearly approximate the true value if the successive settings are made by approaching the match alternately from opposite directions.

The measurements are best carried on by two persons, the observer who makes the color matches and the flicker photometer settings, and an assistant who regulates the currents, reads the scales, and records the readings.

Nothing more need be said of the measurement of T_w and R_w than has already been given under II-9 above.

The following is a standard procedure for the measurement of λ, p, and i:

(a) The currents in the white lamp and the sample lamp being regulated to their standard values by the assistant, and the sample being in place, the observer makes a number of settings (say 10) of the wave-length drum for λ, *each setting being made with separate*

simultaneous readjustments of the wave-length, spectral lamp current, and the distance of the white lamp from the ground glass.[22]

(b) The wave-length drum is then set to the mean of these ten readings and left constant while the measurements of p and i are being made.

(c) The wave-length drum being set to this mean value, the observer matches the field by adjusting the spectral lamp current and the distance of the white lamp from the ground glass.

(d) These settings remaining constant he makes the setting $D_{\Lambda+w}$ of the comparison lamp by the flicker photometer and follows this by the settings D_{Λ} and D_{w} in this order (Cf. particularly II-8 and III-2-c above).

The procedure just outlined under *c* and *d* above is repeated as often as the desired precision demands. A typical record sheet with ten such measurements is shown in Fig. 7, which together with the computations shown on it should be self-explanatory. A determination of Λ, p and i as here shown requires normally from 45 to 60 minutes.

Another procedure for obtaining p and i is to note the reciprocal square of the white lamp distance and the spectral lamp current as set for each of the initial ten readings of Λ. Then set the means of these with the mean Λ and make the flicker analysis to determine p and i repeating it with these constant settings as often as desired. This procedure is much shorter but gives no direct measure of the precision of adjusting the mixture of homogeneous and white light to match the sample.

If p is very high say over 0.8 and T_w or R_w very low, p may be better measured by the method of collimator slit width measurements, without use of the flicker photometer. With the collimator slit at its normal width, the field is matched just as before. The white light is then cut out of the mixture field thereby reducing its brilliance and increasing its saturation. Neglecting the very small saturation difference, the observer then adjusts the collimator slit to make the mixture field match the sample field in brilliance. Call the initial slit width W_0 and the new slit width W_x. Then $p = \frac{W_0}{W_x}$.

[22] If the white component of the mixture field cannot be sufficiently reduced by increasing the distance of the white lamp from the ground glass it may be reduced in large steps by inserting one or more extra plates of similar rough ground glass, using the adjustment of distance as a subsequent fine adjustment.

HOMO-HETERO ANALYSIS OF COLOR STIMULUS
(Apparatus, I.G.P. 1922-23, Room 408 S)
Col. Temp. White Light 5000° K. Material White Standard $MgCO_3$
Colimator slit width 1.0 mμ. Ocular slit width 5.0 mμ
Field size {Color Match, 4°; Flicker, 2°} Field brightness ____

SAMPLE Jena 556 ✓
DATE Aug 9, 1923 OBSERVER I.G.P.
HOUR 9:15 – 10:11 (Start) (Finish) EYE, Right,

DETERMINATION OF DOMINANT WAVELENGTH (Λ)
(Each reading with independent adjustments of brightness and purity)

		Λ	Residuals
Scale Readings (Wavelength Drum)	1	574.8	0.1
	2	574.1	.8
	3	575.1	.2
	4	574.8	.1
	5	574.9	.0
	6	575.1	.2
	7	575.1	.2
	8	574.8	.1
	9	575.1	.2
	10	575.3	.4
Mean		574.91 ✓	.23 ✓
Scale Correction		−1.21 ✓	

True Λ 573.7 ✓ ± .064 ✓ (P.E.)

DETERMINATION OF PURITY (p) WITH Λ CONSTANT AT ABOVE MEAN VALUE, BY ANALYSIS OF MIXTURE BY FLICKER PHOTOMETER SUBSTITUTION METHOD, CONSTANT COMPARISON LAMP. (The readings on each line below are for independent adjustments of brightness and purity.)

	Distances of Comparison Lamp from Milk Glass Screen centimeters			Purity $p = \left(\frac{D_{\Lambda+W}}{D_\Lambda}\right)^2$	Residuals	Impurity $i = \left(\frac{D_{\Lambda+W}}{D_W}\right)^2$	Residuals	p + i
	$D_{\Lambda+W}$ (mixture)	D_Λ (homogeneous)	D_W (white)					
1	16.3	17.3	42.7	0.888 ✓	0.042 ✓	0.146 ✓	0.015 ✓	1.034 ✓
2	16.4	18.2	37.5	.813 ✓	.033 ✓	.172 ✓	.011 ✓	.985 ✓
3	16.4	18.3	38.3	.804 ✓	.042 ✓	.183 ✓	.022 ✓	.987 ✓
4	16.0	17.9	38.1	.799 ✓	.047 ✓	.171 ✓	.011 ✓	.970 ✓
5	15.9	17.6	39.0	.816 ✓	.030 ✓	.166 ✓	.005 ✓	.982 ✓
6	16.5	18.0	39.0	.840 ✓	.006 ✓	.173 ✓	.018 ✓	1.013 ✓
7	16.4	17.5	41.8	.878 ✓	.032 ✓	.154 ✓	.007 ✓	1.032 ✓
8	16.5	17.9	41.7	.850 ✓	.004 ✓	.156 ✓	.005 ✓	1.006 ✓
9	16.4	17.2	46.5	.909 ✓	.063 ✓	.124 ✓	.037 ✓	1.033 ✓
10	16.6	17.9	41.1	.860 ✓	.014 ✓	.163 ✓	.002 ✓	1.023 ✓
	Means			0.8457	.0313	0.1614 ✓	.0133 ✓	1.0071

p = { direct measurement ..... 0.8457 ± 0.0089 ✓
from 1 − i ..... 0.8386 ± 0.0037 ✓
weighted mean ..... 0.8421 ± 0.___ }

FIG. 7. *Specimen Data Sheet.*

V. TESTS OF PERFORMANCE AND ACCURACY

1. PRECISION

Data on the precision of measurement of Λ, p and i for various values of these quantities are shown in Table 1. They give a general idea of the precision under various conditions with this apparatus, but are not sufficiently extensive to warrant any extended discussion, nor to justify the drawing of general conclusions as to sensibility. It will

TABLE 1. *Precision*

p (Approx)	Λ (Approx)	P.E. of a Single Observation of Λ Derived from 10 successive observations each with independent adjustments of purity and brightness.			P.E. of a Single Observation of p Derived from 10 successive adjustments and measurements of p with Λ constant.			P.E. of a Single Observation of i Derived from 10 successive adjustments and measurements of i with Λ constant.		
		IGP	KSG	AEOM	IGP	KSG	AEOM	IGP	KSG	AEOM
0.14	478mμ	±0.65mμ	±0.21mμ	±0.65mμ	±0.009	±0.005	±0.009	±0.028	±0.013	±0.031
		.44	.20	.22	.005	.006	.014	.025	.016	.031
		.71	.39	.29	.010	.007	.009	.012	.014	.027
0.28	539	3.6	2.3	4.0	.013	.008	.015	.012	.014	.013
		5.8	3.5	2.2	.007	.010	.012	.020	.013	.019
		6.3	2.0	3.1	.005	.005	.010	.016	.009	.019
0.28	573	.59	.56	1.0	.010	.020	.019	.020	.019	.019
		.96	.62	.52	.015	.018	.019	.019	.022	.031
		.56	.52	.49	.020	.021	.011	.024	.018	.016
0.26	582	.81	.66	.76	.012	.011	.012	.021	.013	.022
		1.1	.74	.50	.015	.009	.013	.018	.011	.022
		1.1	.95	.32	.011	.016	.011	.019	.015	.018
0.82	574	.24	.26	.14	.024	.016	.031	.015	.011	.014
		.20	.25	.17	.027	.016	.031	.011	.006	.017
		.31	.11	.17	.029	.007	.028	.021	.008	.019
0.84	540	1.2	1.1	1.9	.021	.021	.024	.016	.007	.023
		1.1	1.6	2.7	.018	.015	.022	.011	.016	.024
		1.4	1.3	1.2	.023	.013	.046	.016	.011	.019

be noted that the probable errors of Λ are in fair accord with previous data on hue sensibility to wave-length difference.[23] The probable errors given for p and i necessarily involve errors of photometry in analysis of the mixture as well as the sensibility of compounding the mixture. They are thus not strictly true indices of saturation sensibility. There are no data available on which one can predict the sensibility to be expected in the measurement of purity or impurity for various combinations of Λ and p at various brightnesses.

[23] Jones: Jo. Op. Soc. Am. *1*, pp. 63-77; 1917

2. TEST FOR DIFFUSED HETEROGENEOUS ILLUMINATION IN THE SPECTRUM FIELD AND CORRECTION FOR ITS EFFECT

The spectral lamp contributes to the mixture field not only the homogeneous light indicated by the spectrometer reading, but also, by reason of scattering, a small amount of heterogeneous light of approximately the spectral distribution of the source. To a certain extent this is eliminated by diaframing the path of light through the spectrometer. However, the lenses, the photometric cube, the white light reflection plate and especially the grating film have surfaces which diffuse light to a noticeable degree. These surfaces being in the line of sight and the path of light, it is obvious that a certain component of the brightness of the mixture field is due to heterogeneous light from the spectral lamp. In a rough way we may regard this diffused light as "white" light. It therefore follows that to match the sample field, less white light must be added to the mixture field than would be the case if there were no scattered light from the spectral source; and the measured values of p will be higher than the true values. To obtain the magnitude of this effect it is necessary to measure

$$\frac{B_d}{B_{d+\Delta}}$$

where

B_d is field brightness due to diffuse light from the spectral source, and

$B_{d+\Delta}$ is field brightness due to the diffuse light plus the homogeneous light from the spectral source.

This ratio has been measured and the results are shown in Fig. 8. These measurements were made in terms of field brightness by means of the flicker disk and the comparison lamp in the following way, in which the relative values of B_d and $B_{d+\Delta}$ were obtained by a substitution method in terms of the distance of the comparison lamp from the milk glass.

The 2° diafram of the flicker photometer being set on the mixture field, the light from the white lamp being cut off by a shutter and the spectral lamp being operated at near its maximum current, a setting of the comparison lamp for minimum flicker was made under the following conditions:—

(1) Wave-length setting was 400 mμ.

(2) The collimator slit width was 1.7 mm, i.e. ten times its standard width.

(3) The collimator slit was covered by a piece of "noviol" glass, the transmission of this glass for wave-length 400 mμ being less than 0.001 while its total transmission for the light of the spectral lamp was 0.85.[24]

(4) The reduction filter of transmission 0.1258 was between the comparison milk glass and the flicker disk (See III-4 above).

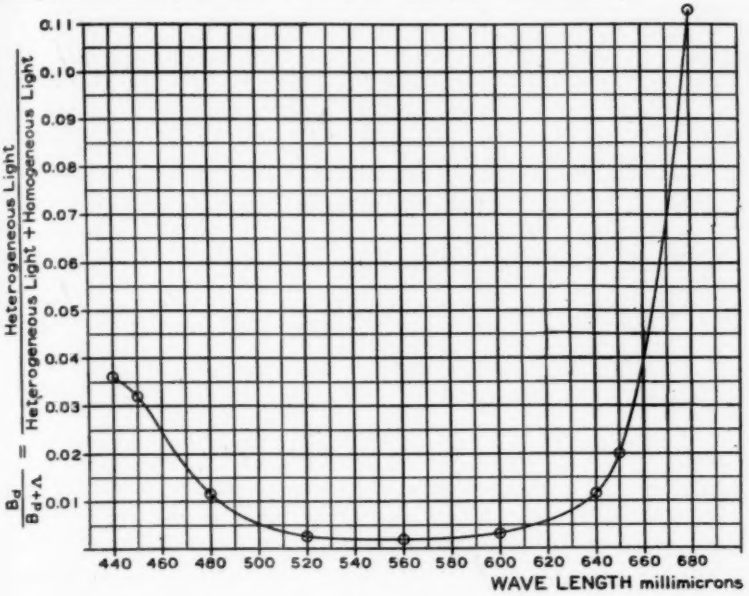

FIG. 8. *Ratio of Heterogeneous Light to Total Light Passing the Spectrum Slit.*

The collimator slit was then set to its standard width, 0.17 mm, the "noviol" glass and the comparison lamp filter were removed, and, all other conditions remaining constant, a series of settings of the comparison lamp for disappearance of flicker was made for a series of values of wave-length (Λ) of light in the mixture field.

Calling the distance of the comparison lamp from the milk glass under the first conditions, D_d and its distance under the new conditions $D_{d+\Lambda}$ (a function of Λ) it is obvious that

$$\frac{B_d}{B_{d+\Lambda}} = \frac{0.1258}{10 \times 0.85}\left(\frac{D_{d+\Lambda}}{D_d}\right)^2$$

[24] Computed from measured spectral transmission of the glass and assumed spectral distribution of the lamp.

This ratio is a function of λ and is so plotted in Fig. 8.

Neglecting several minor considerations and second order effects, it is obvious that, to as close an approximation as is justified under the circumstances,

$$p = p_m(1-I)$$

where

p is true purity;

p_m is measured purity; and

I is $\dfrac{B_d}{B_{d+\lambda}}$

In many practical measurements this correction may be neglected, because, in most usual natural and artificial samples λ will be found to lie between 480 and 640 millimicrons. (Cf. Fig. 8.).

This error may be eliminated experimentally by the use of suitable selective filters over the collimator slit.

3. TEST OF THE INVERSE SQUARE LAW FOR THE AUXILIARY PHOTOMETER

In the design of the auxiliary photometer particular care was taken to insure from *a priori* considerations that the brightness of the side of the milk glass toward the flicker disk should vary strictly inversely as the square of the distance of the comparison lamp from the milk glass. That this condition was actually fulfilled was afterward verified by way of an independent calibration of the filter used to extend the scale of the auxiliary photometer (See III-4 above).

The total transmission of this filter for the comparison light was computed from its spectral transmission as independently measured by visual and radiometric methods.[25] The following check values were found:—

Transmission from inverse square law = 0.1258

Transmission from spectral transmission = 0.1260.

4. TEST OF THE ADDITIVITY OF HOMOGENEOUS AND WHITE BRIGHTNESSES[26]

In what has preceded it has been tacitly assumed that

$$B_\lambda + B_w = B_m$$

where

B_λ is the brightness of the mixture field due to the homogeneous light alone;

[25] Measurements by K. S. Gibson and M. Katherine Frehafer, Bureau of Standards.

[26] Cf. H. E. Ives, Phil. Mag. Dec., 1912, pp. 845-853.

B_w is the brightness of the mixture field due to the white light alone; B_m is the brightness due to their mixture.

This assumption may be stated in another form in the equation

$$p+i=1$$

The question now arises as to whether or not the values of p and i as actually determined by the method of analysis here adopted satisfy this condition.

Table 2 gives a synopsis of most of the data which we have on this question. Other data not presented are in substantial accord with these. It should be observed that each value of $p+i$ tabulated here is derived from the mean of ten measurements of p and i carried out in the form shown in Fig. 7. Values of $p+i$ from a single measurement each of p and i, occasionally depart from unity by as much as 0.05 or (*very rarely*) even more than this. Such large departures may probably be attributed to blunders rather than a true lack of precision. In considering Table 2, it should be noted that the data

TABLE 2. *Experimental Values of* $p+i$. Each value of p+i is the MEAN from 10 successive measurements of p and i.

Approximate p →	0.14	0.26	0.28	0.28	0.82	0.84	
Approximate A →	478	582	573	539	574	540	
Obs.							
↓	1.010	0.998	1.005	0.991	0.984	0.977	
IGP	1.043	.990	1.001	.984	1.007	.973	
	1.008	.984	.976	.992	.992	.977	
Mean	1.020	0.991	0.994	0.989	0.994	0.976	0.994
KSG	1.027	1.019	1.005	1.002	0.993	0.997	
	1.005	.998	0.999	1.011	1.003	1.002	
	.999	.995	1.005	1.004	.996	.999	
Mean	1.010	1.004	1.003	1.006	.997	.999	1.003
AEOM	1.009	0.998	1.009	1.004	1.012	0.964	
	0.989	1.042	1.027	1.002	0.992	.990	
	0.976	1.007	0.991	1.001	1.000	.972	
Mean	0.991	1.016	1.009	1.002	1.001	0.975	0.999
Mean of indiv. means	1.007	1.004	1.002	0.999	0.997	0.983	0.999

in the first and last columns were obtained under relatively unfavorable conditions of field brightness, and should not be given as much weight as the other data. The observer felt that the brightness was too low for satisfactory work.

Interpreting these data in the light of personal acquaintance with the experimental conditions, the following conclusion is drawn:—

Provided that the field brightnesses are such that the measurements may be made with comfort and satisfaction to the observer, brightnesses due respectively to homogeneous (spectral) stimuli and heterogeneous stimuli (white light) as measured by the substitution flicker method are strictly additive.

That is, to an approximation comporting with the accuracy of the measurements, it is *experimentally* true that

$$p+i=1$$

or

$$\frac{B_w}{B_m}+\frac{B_A}{B_m}=1$$

or

$$B_w+B_A=B_m$$

where B_w, B_A and B_m are measured by the substitution flicker method.

It remains to qualify this conclusion by a brief statement of the scope and limitations of the experimental data on which it is based:—

(1) The range of wave-length of the homogeneous light for which the tests have been made is from about 480 to 580 millimicrons.

(2) The range of values of purity over which the tests have been made is from about 0.15 to 0.85.

(3) The diameter of the field which is circular subtends an angle of 2° at the observer's eye.

(4) The surrounding field is dark.

(5) The comparison light has a spectral distribution *about* that of a Planckian radiator at a temperature of 2300° K. (On account of the slightly selective transmission of the milk glass, the color is not *strictly* Planckian; but the image of the milk glass in the monel metal mirror of the flicker disk is *approximately* matched by a Planckian radiator at 2300° K.)

(6) As regards the accuracy, it appears certain that the departure of $p+i$ from unity can not be greater than ± 0.005, and the data would seem to indicate that the departure is probably less than this.

BUREAU OF STANDARDS,
WASHINGTON,
SEPT. 12, 1923.

SOME TESTS OF THE PRECISION AND RELIABILITY OF MEASUREMENTS OF SPECTRAL TRANSMISSION BY THE KOENIG-MARTENS SPECTROPHOTOMETER[1]

By Irwin G. Priest, H. J. McNicholas, and M. Katherinė Frehafer

ABSTRACT

The Koenig-Martens spectrophotometer as installed and used at the Bureau of Standards has been tested by measuring the transmissions of rotating sectored disks of known angular aperture. Measurements of the spectral transmission of blue filters by different observers have also been compared. The photometric probable error at various wave-lengths has been computed from the observations of three different observers.

From the data, which are presented in considerable detail in tables and graphs, the following conclusions are drawn:

1. For heterogeneous illumination, the probable error of a single observation (involving one reading each of θ_1 and θ_2) is about seven tenths of one percent of the transmission when measurements are made at the field brightness most favorable to accurate matching.

2. For homogeneous illumination the probable error of a single observation of transmission 0.37 is nearly constant and equal to about one percent of the transmission in the case of the following wave-lengths of mercury and helium: mercury, 435.8, 546.1 mμ; helium, 447.2, 492.2, 501.6, 667.8, 706.5 mμ.

3. For heterogeneous illumination, the actual error[2] of the mean of ten observations for transmissions even as low as 0.015 and for wave-lengths between 430 and 700 mμ is usually less than one percent of the transmission, provided the transmission is not changing rapidly with wave-length.

4. There is no systematic error of measured transmission with respect to wave-length.

I. Introduction

The question of the accuracy attainable in visual spectrophotometry is one of very considerable importance in colorimetry in connection with the proposal to specify colors by way of a spectrophotometric specification of their actual stimuli.[3] There can be no doubt that

[1] Paper presented at Eighth Annual Meeting Optical Society of America, Cleveland, Ohio, October 26, 1923.

Published by permission of the Director of the Bureau of Standards, U. S. Department of Commerce, and the Director of the Munsell Research Laboratory. These tests were made incidental to the cooperative work on color standardization by the Bureau of Standards and the Munsell Color Company.

[2] Difference between measured value and true value as given by the angular calibration of the sectored disk.

[3] "A spectrophotometric table, derived from at least 25 points (for a continuous spectrum) gives the only unique description of a color, and it appears probable to the writer that the requirements of precision technical color measurement are most likely to be met by the development of simple and rapid means of plotting and recording accurate spectrum plots of reflection or transmission characteristics."—H. E. Ives, Jo. Frank. Inst. p. 700; Dec. 1915. Cf. also:—Troland: "Report of Colorimetry Committee," J. O. S. A. and R. S. I. *6*, pp. 574-575; 1922 and Priest: Trans. I. E. S. *13*, p. 41; 1918.

in many cases such a form of specification is highly desirable. Aside from the question of adequate sensibility and reliability, such a specification of color is certainly the most fundamental that can be given in the nature of the case. However the following very pertinent question arises at once: *May a difference in spectral distribution of stimuli less than could be certainly determined by spectrophotometric methods still be large enough to cause a difference in color?*[4]

This question in its general aspect is so intricate that there is little basis for its profitable discussion at present. It comprises two questions, viz., (1) That of the relative sensitiveness of the color-match method[5] and the spectrophotometric method; (2) That of the independent reproducibility of spectrophotometric data.

The relative sensibility of the two methods will not be treated in this paper, which is limited to the statement of the results of a few tests of the precision and accuracy of spectrophotometric measurements.

In a general way the precision of spectrophotometric measurements may be inferred from the data of Koenig and Brodhun.[6] However, when all sources of error are considered, all who are concerned with them must feel considerable doubt as to just how much confidence may be placed in a "spectrophotometric curve." Indeed this question must be settled independently for each independent set of conditions under which such measurements are made. The tests to be reported here were undertaken to settle this question for one such particular case. There was originally no intention to make an independent publication of them *per se*; but since there appears to be a notable lack of data on which those interested may form an opinion of the accuracy attainable in such measurements, it was suggested to the authors that it would be worth while to publish these results as indicating at least what had actually been done under certain particular circumstances.[7] It is hoped that this paper will be rendered *passé* at the earliest possible date by an extensive and reasonably complete

[4] "Color" being understood as "sensation." Cf. "Report of Colorimetry Committee," J. O. S. A. and R. S. I., *6*, pp. 531-534; August, 1922.

[5] Matching of an unknown color by admixture of other known stimuli, as in trichromatic analysis, monochromatic analysis, and other methods.

[6] Arthur Koenig's Gesammelte Abhandlungen zur Physiologischen Optik, (Leipzig) pp. 116-139; 1903.

[7] Most of these tests were made in March, 1922. When the results were subsequently shown to Dr. A. H. Pfund in response to his inquiry as to the accuracy of our spectrophotometric work, he suggested that they ought to be made more generally available by publication.

discussion of the precision and accuracy of spectrophotometry and spectroradiometry in general, as well as their bearing on color specifications.

II. DESCRIPTION OF EXPERIMENTAL WORK

These tests were made with the Koenig-Martens[8] spectrophotometer under the following conditions:—

1. NATURE OF TESTS MADE

The experimental tests consisted primarily in the measurement of the spectral transmission of:—

(a) four rotating sectored photometric disks of apertures known from angular calibration;

(b) two blue glass filters of Ives-Brady daylight glass.[9]

The experimental data are here to be considered in the following aspects:—

(a) With regard to the data on sectored disks we shall report for different observers:

(α) the photometric probable errors at different wave-lengths which are to be regarded merely as indices of the precision of photometric matching;

(β) the differences between the measured values of transmission and the true values as given by the angular calibration on the assumption of the Plateau-Talbot law.

(b) With regard to the data on the blue filters we shall show:

(α) for one filter, the photometric probable errors at various wave-lengths for two observers.

(β) for two different filters, the degree of agreement of different observers and separate determinations of the same observer.

Further details will be evident from an examination of the accompanying tables and graphs.

2. METHOD OF ILLUMINATION[10]

The photometric field was illuminated by the light from a uniformly illuminated plane diffusing surface about one meter from the collimator

[8] Ann. der Phy. (4) 12 pp. 984-1003; 1903. The instrument used in the present work was purchased from Schmidt & Haensch, the makers, about 1912.

[9] E. J. Brady. Trans. I. E. S. *9*, pp. 937-952; 1914.
H. E. Ives. Jour .Frank. Inst. *177*, p. 496; 1914.

[10] The illumination apparatus of which the essential features are specified here, was that which has been continuously used with the Koenig-Martens spectrophotometer in the Colorimetry Section of the Bureau of Standards for about five or six years. Cf. B. S. Tech. Pap. *148*, p. 5; B. S. Tech Pap. *167*, p. 20; and B. S. Sci. Pap. *440*, pp. 126-127 with Fig. 1.

slit, the collimator being pointed in a direction normal to this surface. *The only optical parts or media other than air between the slit and this surface were the totally reflecting rhombs before the slit* (q_1 and q_2, Fig. 6, p. 991, Ann. der Phy., 4th series, *12;* 1903), *and the plane neutral plate glass windows of the lamp box* (*vide infra*).

In the case of the heterogeneous light and the homogeneous mercury light this surface was of magnesium carbonate and was illuminated by lamps inclosed in a white lined rectangular box occupying the space between the surface and the slit, the carbonate block being, in effect, a part of the box wall and the light reflected from it passing to the above mentioned rhombs by way of two small holes covered by neutral glass in the box wall opposite the block. A series of diaphragms inside the lamp box prevented illumination of the slits by light other than that from the magnesium carbonate surface.

In the case of the homogeneous helium lights a ground glass plate was substituted for the magnesium carbonate block and was illuminated by grid shaped helium lamps in a white lined box on the side of the ground glass opposite the spectrophotometer.

Ten 600-watt gas-filled tungsten lamps were used in the box for heterogeneous illumination. The heterogeneous brightness of the magnesium carbonate surface was then about five lamberts.[11] No measurements of the homogeneous brightness for mercury and helium sources were attempted. With these sources, however, since the slits could be opened wide, the homogeneous field brightness, for the wave-lengths used, was as large as or larger than the corresponding brightness obtained with the heterogeneous source and narrow slits.

3. CONDITIONS OF OBSERVATION

The eyes of the observer making the photometric match were completely shaded from extraneous light. The photometric field was substantially perfect with regard to the disappearance of the division line for equality of brightness.

4. METHOD OF MEASUREMENT

The "sample" (which in some cases was a rotating sector disk, and in others a blue glass) was placed in the space between the rhombs (vide supra) and the lamp box. For each wave-length setting, five successive

[11] This is about one half the brightness of magnesium carbonate illuminated by the noon summer sun in a clear sky. It may be noted that much higher brightnesses are readily available for transmission measurements. For example, using a *small* white lined box containing two 400-watt lamps, it is possible to obtain a brightness of the order of *one hundred*

settings of the ocular nicol to match the field in brilliance were made with the sample so placed as to reduce the illumination in one half of the field; then ten similar settings with it placed to reduce the illumination in the other half; and finally five more with it again reducing the illumination in the first half.

The transmission was then computed by the formula

$$T = \tan \theta_2 \cot \theta_1$$

where θ_1 is the scale reading of the ocular nicol for match when the sample is covering the half of the field which is extinguished for $\theta =$ zero.

θ_2 is the scale reading of the ocular nicol for match when the sample is covering the half of the field which is extinguished for $\theta = 90°$, θ being the general symbol for the ocular nicol scale reading.[12]

III. RESULTS AND CONCLUSIONS

The experimental results are shown in Tables 1, 2, 3, 4, and 5, and Figs. 1 and 2. These tables and graphs are self-explanatory; and it is thought that detailed comment by the authors would be superfluous to those who are interested in such data. However, attention is invited to the agreement between the measurements with heterogeneous and homogeneous illumination in the case of the blue filters (Figs. 1 and 2). Because of the question of spectral purity raised by the finite slit widths with heterogeneous illumination, it has been our uniform custom for several years to check the determinations with heterogeneous illumination by independent observations with homogeneous mercury and helium illumination. This question is, of course, only important in case the transmission changes rapidly with wave-length. The distribution of mercury and helium wave-lengths usually permits of a check on any steep part of a spectral transmission curve.

On the basis of the data presented, we draw the following conclusions:

1. For heterogeneous illumination, the probable error of a single observation (involving one reading each of θ_1 and θ_2) is about seven tenths of one percent of the transmission when measurements are made at the field conditions most favorable to accurate matching.

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lamberts. Such a source, which we are now installing, will greatly facilitate measurements for the longer and shorter wave-lengths and for samples of very low transmission. Such intense illumination is not feasible for reflection measurements.

[12] Cf. Ann. der Phy. (4) *12*, p. 997; 1903

2. For homogeneous illumination, under the conditions of illumination which we have used, the probable error of a single observation of transmission 0.37 is nearly constant and equal to about one percent of

TABLE 1.

B. S. Sectored Disk #8 (Six Slots)
True Transmissions from Angular Calibration by B. S. Div. of Weights and Measures
Slots 1+3+5=0.0078
Slots 3+4+6=0.0076
Total Transmission of Disc=0.0154± .0002

Wave Length in millimicrons	Transmissions measured on Spectrophotometer		
	T for slots 1+3+5 by M. K. F.	T for slots 2+4+6 by M. K. F.	T for 6 slots by R. H. S.
435.8	.00775	.00759	.01518
546.1	.00772	.00766	.01523
578.0			.01510
587.6	.00776	.00749	
650.0	.00772	.00749	.01525

TABLE 2.

Sectored Disk, B. S. 30547
True transmission=0.1009 from Angular Calibration by B. S. Division of Weights and Measures

Wave Length in millimicrons	T by I. G. P.
520	0.1006
	0.1007
580	0.1002
600	0.1009
	0.1009
620	0.1008
	0.0996

the transmission in the case of the following wave-lengths of mercury and helium:

Mercury, 435.8, 546.1 mμ;
Helium, 447.2, 492.2, 501.6, 667.8, 706.5 mμ.

3. For heterogeneous illumination the actual error[13] of the mean of ten observations for transmissions even as low as 0.015 and for wave-

[13] Difference between measured value and true value as given by the angular calibration of the sectored disk.

lengths between 430 and 700 mμ is usually less than one percent of the transmission, provided the transmission is not changing rapidly with wave-length.

4. There is no systematic error of measured transmission with respect to wave-length.

TABLE 3.
Sectored Disk, B. S. No. 22143 B, 4 and 3
$T_d = 0.1029$
Heterogeneous Illumination

Observer					I. G. P.			H. J. M.		
	Collimator Slit Width		Ocular Slit Width							
Wave-length in mμ	mechanical in mm	spectral in mμ	mechanical in mm	spectral in mμ	T_m	$\frac{T_m - T_d}{T_d}$	$\frac{(P.E.)_{T_m}}{T_m}$	T_m	$\frac{T_m - T_d}{T_d}$	$\frac{(P.E.)_{T_m}}{T_m}$
430	0.5	3.8	0.3	2.3	0.1027	−0.002	±0.019			
					.1045	+ .016	.016			
460	0.3	3.1	0.2	2.1				0.1028	−0.001	± .0076
								.1016	− .013	.0096
490	0.1	1.3	0.2	2.6				.1020	−..009	.0043
520	0.1	1.7	0.2	3.4	.1021	−0.008	0.0038	.1034	+ .005	.0032
					.1041	+0.012	0.0023			
560	0.1	2.2	0.2	4.4				.1033	+ .004	.0024
600	0.1	2.8	0.2	5.6				.1036	+ .007	.0030
640	0.1	3.5	0.2	7.0				.1035	+ .006	.0036
660	0.2	3.8	0.2	3.8				.1029	.000	.0047
700	0.3	13.8	0.2	9.2				.1046	+ .017	.0075
	0.5	23.0	0.2	9.2	.1042	+0.013	0.0051			
	0.5	23.0	0.2	9.2	.1025	−0.004	0.0071			
				Alg. mean		+0.005			+0.002	

T_d is true transmission of the disk from angular calibration by B. S. Division of Weights and Measures.

T_m is transmission as measured on the Koenig-Martens spectrophotometer, each tabulated value being the mean of ten observations.

$(P.E.)_{T_m}$ is the probable error of the mean as derived from the ten residuals of θ_1 and θ_2 by propagation of error formulas.

In more general terms we conclude:

1). Under suitable conditions of illumination and observation the precision of measurement with the Koenig-Martens spectrophotometer

TABLE 4
Sectored Disk, B.S. No. 27692
$T_d = 0.3679$
(Heterogeneous Illumination)

Observer					I. G. P.			M.K.F.		
	Collimator Slit Width		Ocular Slit Width							
Wavelength mμ	mechanical in mm	spectral in mμ	mechanical in mm	spectral in mμ	T_m	$\frac{T_m - T_d}{T_d}$	$\frac{(\text{P.E.})_{T_m}}{T_m}$	T_m	$\frac{T_m - T_d}{T_d}$	$\frac{(\text{P.E.})_{T_m}}{T_m}$
450					0.3719	+0.011	±0.0046	0.3640	−0.011	±0.0051
								.3685	+0.0016	.0062
470	0.1	1.1	0.2	2.2	0.3645	−0.0092	±0.0068			
	0.1	1.1	0.2	2.2	.3700	+0.0057	.0084			
500	0.1	1.4	0.2	2.8	.3657	−0.0060	.0032			
	0.1	1.4	0.2	2.8	.3668	−0.0030	.0034			
520					.3691	+0.0031	.0034	0.3669	−0.0027	0.0030
					.3680	+0.0003	.0022	.3695	+ .0043	.0032
580	0.1	2.5	0.2	5.0	.3690	+0.0030	.0023			
	0.1	2.5	0.2	5.0	.3693	+0.0038	.0025			
600					.3690	+0.0030	.0037	0.3651	−0.0076	0.0010
					.3679	.0000	.0035	.3695	+ .0043	.0016
680					.3632	−0.013	.0031			
700	0.3	13.8	0.2	9.2	.3640	−0.011	.0028			
	0.3	13.8	0.2	9.2	.3589	−0.024	.0038			
710					.3685	+0.0016	.0052			
					(Homogeneous Illumination)					
435.8 Hg	Wide		Wide		0.3699	+0.0054	±0.0034			
447.2 He	"		"		.3644	− .0092	.0038			
492.2 He	"		"		.3693	+ .0038	.0033			
501.6 He	"		"		.3678	− .0003	.0034			
546.1 Hg	"		"		.3684	+ .0015	.0026			
667.8 He	"		"		.3623	− .0153	.0032			
706.5 He	"		"		3653	− .0071	.0045			

T_d is the true transmission of the disk from angular calibration, by B. S. Division of Weights and Measures.

T_m is the transmission as measured on the Koenig-Martens spectrophotometer, each tabulated value being the mean of ten observations.

(P.E.) T_m is the probable error of the mean as derived from the ten residuals of θ_1 and θ_2 by propagation of error formulas.

TABLE 5. *Ives-Brady Daylight Glass* (Submitted by E. J. Brady letter of Dec. 2, 1921)

Particular care was taken always to use same part of specimen (see Fig. 1).

Each tabulated value of transmission (T) is the mean of ten observations. The probable errors $(P.E.)_T$ are the probable errors of these means as derived from the ten residuals of θ_1 and θ_2 (see appendix) by propagation of error formulas.

Wave Length mμ	I. G. P., March 30-31, 1922						M. K. F., March 31, 1922						I. G. P., April 3-4, 1922					
	Collimator Slit Width		Ocular Slit Width		T	$\frac{(P.E.)_T}{T}$	Collimator Slit Width		Ocular Slit Width		T	$\frac{(P.E.)_T}{T}$	Collimator Slit Width		Ocular Slit Width		T	$\frac{(P.E.)_T}{T}$
	mechanical mm	spectral mμ	mechanical mm	spectral mμ			mechanical mm	spectral mμ	mechanical mm	spectral mμ			mechanical mm	spectral mμ	mechanical mm	spectral mμ		
**435.8	wide		wide		0.6381	0.0025	0.5		0.2		0.6375	0.0039	wide		wide		0.6320	0.0060
440.0													0.4	3.6	0.2	1.8	.5964	.0064
**447.2	wide		wide		.5681	.0043	.5		.2		.5596	.0054	wide		wide		.5490	.0068
450.0													.4	3.8	.2	1.9	.5455	.0034
460.0	0.3	3.0	0.2	2.0	.4828	.0043	.2	2.0	.2	2.0	.4851	.0062	.4	4.0	.2	2.0	.4854	.0046
470.0	.3	3.3	.2	2.2	.4336	.0035	.2	2.2	.2	2.2	.4400	.0067	.2	2.2	.2	2.2	.4420	.0039
480.0	.3	3.6	.2	2.4	.3849	.0035	.2	2.4	.2	2.4	.3868	.0060	.2	2.4	.2	2.4	.3850	.0046
490.0	.3	4.0	.2	2.6	.3321	.0022	.2	2.6	.2	2.6	.3378	.0039	.2	2.6	2	2.6	.3438	.0031
**492.2	wide		wide		.3301	.0057	.5		.2		.3320	.0050	wide		wide		.3199	.0065
500.0	.1	1.4	.2	2.8	.2979	.0015	.2	2.8	.2	2.8	.3005	.0034	.1	1.4	.2	2.8	.3009	.0030
**501.6	wide		wide		.2930	.0038	.5		.2		.3008	.0030	wide		wide		.2920	.0046
510.0	1	1.6	.2	3.2	.2645	.0011	.2	3.2	.2	3.2	.2639	.0043	.1	1.6	.2	3.2	.2661	.0022
520.0	1	1.7	.2	3.4	.2349	.0021	.2	3.4	.2	3.4	.2352	.0037	.1	1.7	.2	3.4	.2374	.0020
530.0	1	1.8	.2	3.6	.1970	.0025	.2	3.6	.2	3.6	.2006	.0041	.1	1.8	.2	3.6	.1998	.0023
540.0	.1	1.9	.2	3.8	.1719	.0014	.2	3.8	.2	3.8	.1722	.0035	.1	1.9	.2	3.8	.1726	.0015
**546.1	wide		wide		.1630	.0018	.5		.2		.1643	.0032	wide		wide		.1640	.0021
550.0	.1	2.1	.2	4.2	.1627	.0032	.2	4.2	.2	4.2	.1644	.0029	.1	2.1	.2	4.2	.1646	.0015
560.0	1	2.2	.2	4.4	.1668	.0018	.2	4.4	.2	4.4	.1661	.0043	.1	2.2	.2	4.4	.1682	.0026
570.0	1	2.4	.2	4.8	.1567	.0019	.2	4.8	.2	4.8	.1607	.0026	.1	2.4	.2	4.8	.1585	.0022
580.0	1	2.5	.2	5.0	.1351	.0028	.2	5.0	.2	5.0	.1381	.0028	.1	2.5	.2	5.0	.1375	.0012
**587.6	wide		wide		.1147	.0017	.5		.2		.1177	.0023	wide		wide		.1163	.0026
590.0	1	2.7	.2	5.4	.1127	.0009	.2	5.4	.2	5.4	.1127	.0031	.1	2.7	.2	5.4	.1137	.0013
600.0	.1	2.8	.2	5.6	.1081	.0041	.2	5.6	.2	5.6	.1074	.0029	.1	2.8	.2	5.6	.1105	.0024
610.0	.1	3.0	.2	6.0	.1011	.0010	.2	6.0	.2	6.0	.1010	.0039	.1	3.0	.2	6.0	.1014	.0012
620.0	1	3.1	.2	6.2	.0884	.0020	.2	6.2	.2	6.2	.0888	.0038	.1	3.1	.2	6.2	.0889	.0011
630.0	.1	3.3	.2	6.6	.0743	.0019	.2	6.6	.2	6.6	.0762	.0038	.1	3.3	.2	6.6	.0754	.0027
640.0	.1	3.5	.2	7.0	.0661	.0032	.2	7.0	.2	7.0	.0670	.0038	.1	3.5	.2	7.0	.0674	.0049
650.0	.1	3.6	.2	7.2	.0701	.0048	.2	7.2	.2	7.2	.0696	.0071	.1	3.6	.2	7.2	.0703	.0033
660.0	.1	3.8	.2	7.6	.0859	.0032	.2	7.6	.2	7.6	.0876	.0077	.1	3.8	.2	7.6	.0865	.0038
**667.8	wide		wide		.1110	.0049	.5		.2		.1151	.0029	wide		wide		.1113	.0050
670.0	.1	4.0	.2	8.0	.1220	.0032							.2	8.0	.2	8.0	.1222	.0023
680.0	.2	8.4	.2	8.4	.1778	.0050							.2	8.4	.2	8.4	.1775	.0034
690.0	2	8.8	.2	8.8	*.2519	.0052							.4	17.6	.2	8.8	*.2431	.0038
700.0	.2	9.2	.2	9.2	*.2901	.0036							.4	18.4	.2	9.2	*.2894	.0056
**706.5	wide		wide		.2982	.0065	.5		.3		.3082		wide		wide		.2913	.0120
710.0	.2	9.6	.2	9.6	*.3072	.0077							.4	19.2	.2	9.6	*.3090	.0060

** Homogeneous Light

*Red filter used over ocular to eliminate stray light

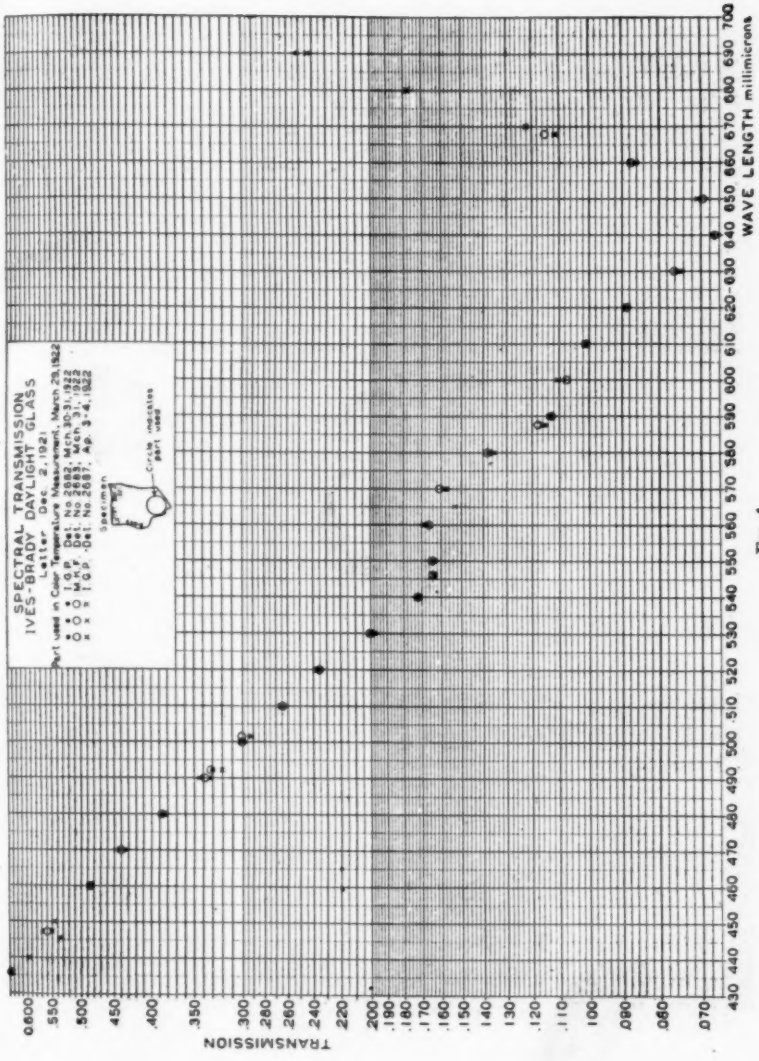
SPECTRAL TRANSMISSION
IVES-BRADY DAYLIGHT GLASS
Letter Dec. 2, 1921
Part used on Color Temperature Measurement, March 29, 1922
Det. No 2882, Mch 30-31, 1922
Det. No 2883, Mch 31, 1922
Det. No 2887, Ap 3-4, 1922
Specimen
Circle indicates part used
TRANSMISSION
WAVE LENGTH millimicrons

FIG. 1

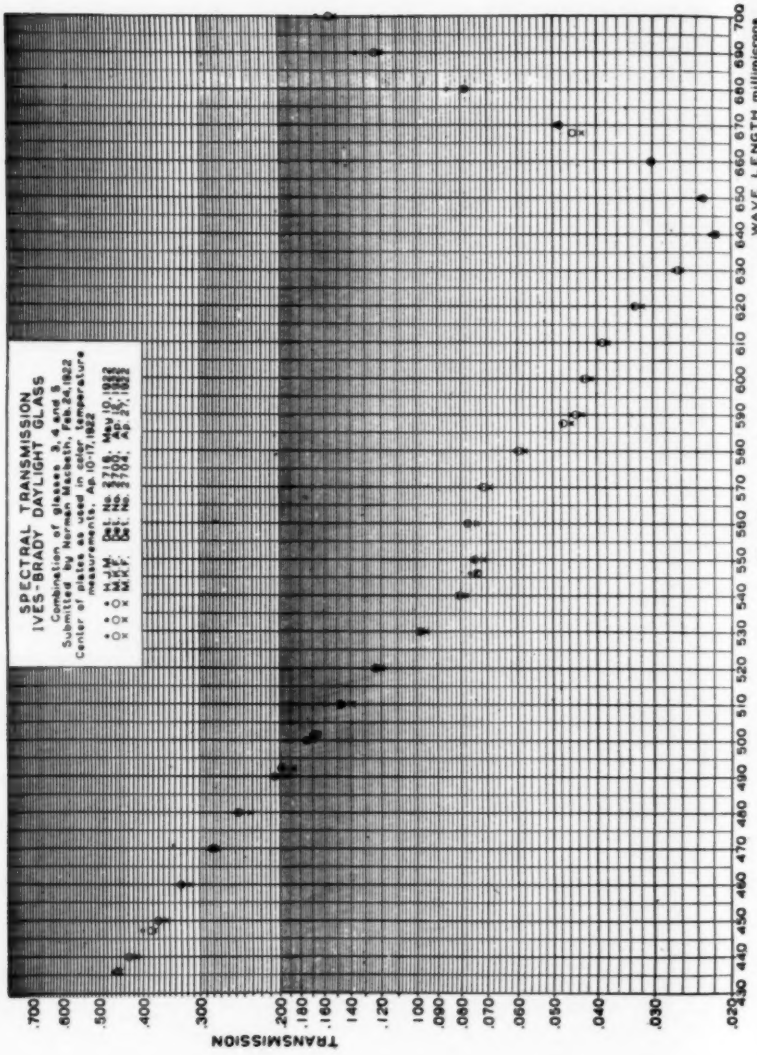
SPECTRAL TRANSMISSION
IVES-BRADY DAYLIGHT GLASS
Combination of glasses 3, 4 and 5
Submitted by Norman Macbeth, Feb. 24, 1922
Center of plate as used in color temperature measurements, Ap. 10-17, 1922
H.V.P. Batt. No. 5708, May 10, 1922
H.V.P. Batt. No. 5702, Ap. 25, 1922
M.V.P.
TRANSMISSION
WAVE LENGTH millimicrons

FIG. 2

is about the same as photometric precision in general under the best possible conditions.

2). Transmissions measured on the Koenig-Martens spectrophotometer in proper adjustment are not subject to a systematic error with respect to wave-length.

3). The necessarily inherent instrumental errors of the Koenig-Martens spectrophotometer used under suitable conditions are less than the general photometric errors of observation.

In conclusion it is worth while to emphasize that the accuracy attained in these tests is only possible under the best conditions by careful work; and is dependent upon the averaging of a number of observations (in this case, ten) for each wave-length. Spectral transmissions dependent upon only one or two observations or made with insufficient field illumination, or by any but the most careful procedure with due consideration of possible sources of error will be of little value in the specification of color.

BUREAU OF STANDARDS,
WASHINGTON, D. C.,
AUGUST, 1923.

WAVE LENGTH millimicrons

Fig. 2

APPENDIX

Formula and Graph for Finding Probable Error of T from the Residuals of Ten Observations each of θ_1 and θ_2 when T is Computed by the Formula $T = \cot\ \theta_1 \tan\ \theta_2$

If $(P.E.)_T$ be the probable error of T when T is computed by the formula $\cot \theta_1 \tan \theta_2$, where θ_1 and θ_2 are each means of ten observations, it may be shown, by the propagation of error and the application of Peters' Approximation Formula,[14] that

$$\frac{(P.E.)_T}{T} = 0.001\sqrt{\frac{[\Sigma(\Delta\theta_1)]^2}{\sin^2 2\theta_1} + \frac{[\Sigma(\Delta\theta_2)]^2}{\sin^2 2\theta_2}}$$

where

$\Sigma(\Delta\theta_1)$ is sum of the ten residuals of θ_1 without regard to sign; and $\Sigma(\Delta\theta_2)$ is sum of the ten residuals of θ_2 without regard to sign.

[14] J. W. Mellor: "Higher Mathematics," 2nd Ed., p. 516.

The graph shown in Fig. 3 will be found useful in computing $\frac{(P.E.)_T}{T}$ for given values of θ_1, θ_2, $\Sigma(\Delta\theta_1)$ and $\Sigma(\Delta\theta_2)$.

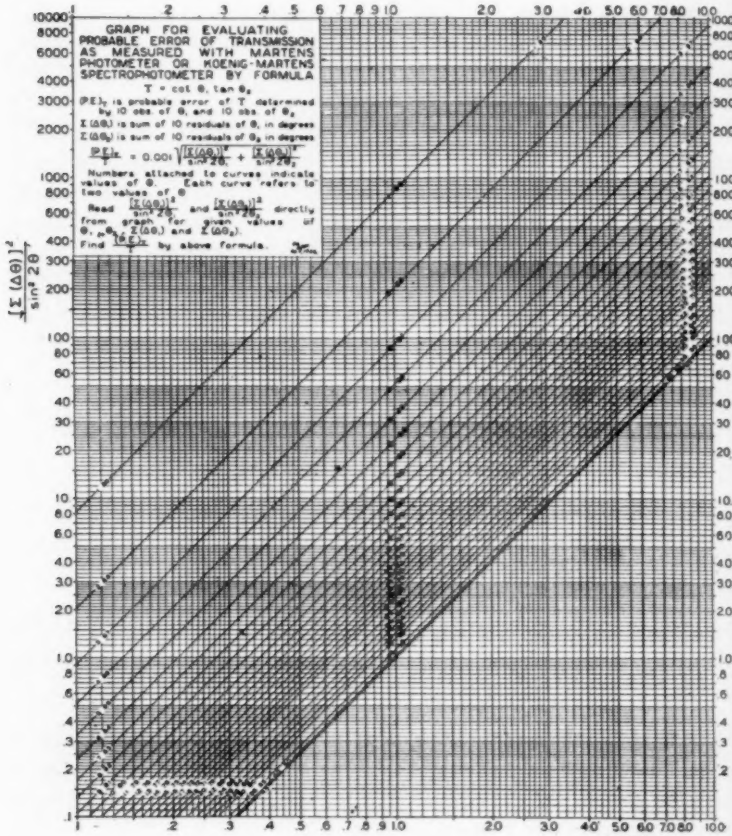
GRAPH FOR EVALUATING PROBABLE ERROR OF TRANSMISSION AS MEASURED WITH MARTENS PHOTOMETER OR KOENIG-MARTENS SPECTROPHOTOMETER BY FORMULA
$T = \cot\theta_1 \tan\theta_2$
$(P.E.)_T$ is probable error of T determined by 10 obs. of θ_1 and 10 obs. of θ_2
$\Sigma(\Delta\theta_1)$ is sum of 10 residuals of θ_1 in degrees
$\Sigma(\Delta\theta_2)$ is sum of 10 residuals of θ_2 in degrees
$\frac{P.E._T}{T} = 0.001\sqrt{\frac{[\Sigma(\Delta\theta_1)]^2}{\sin^2 2\theta_1} + \frac{[\Sigma(\Delta\theta_2)]^2}{\sin^2 2\theta_2}}$
Numbers attached to curves indicate values of θ. Each curve refers to two values of θ
Read $\frac{[\Sigma(\Delta\theta_1)]^2}{\sin^2 2\theta_1}$ and $\frac{[\Sigma(\Delta\theta_2)]^2}{\sin^2 2\theta_2}$ directly from graph for given values of θ_1, θ_2, $\Sigma(\Delta\theta_1)$ and $\Sigma(\Delta\theta_2)$.
Find $\frac{P.E._T}{T}$ by above formula.
$\frac{[\Sigma(\Delta\theta)]^2}{\sin^2 2\theta}$
$\Sigma(\Delta\theta)$ cir. deg

FIG. 3

Experience has shown that, in general, values computed by this formula agree well with the probable errors found by computing ten values of T from the ten values of θ_1 and θ_2 in pairs, and then computing the probable error from the ten residuals of T.